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UNIT - I LINEAR PROGRAMMING PROBLEM (LPP)

Linear means that all the relationship involved in a particular Problem are of degree one.

means to find the values of unknown variables.

Programming means planning.

Ex:- $10x + 8y = 7$

LPP:- Linear Programming is a technique for determining an optimum schedule of Interdependent activities in view of the available resources.

General Linear Programming Problem:- The General linear Programming Problem is given by

$$\text{Max or Min } Z = C_1x_1 + C_2x_2 + \cdots + C_nx_n \rightarrow (1)$$

$$\left.\begin{array}{l} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n \le \text{ or } \ge \text{ or } = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n \le \text{ or } \ge \text{ or } = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \cdots + a_{mn}x_n \le \text{ or } \ge \text{ or } = b_m \end{array}\right\} - (2)$$

$$\text{and } x_j \ge 0 \quad j = 1, 2 \ldots n \rightarrow (3)$$

eq (1) is called objective function which is to be maximized or minimized.

eq (2) is called the set of constraints which is in the form of inequalities or equalities.

eq (3) is called non-negative restrictions.

∴ LPP can be defined an objective function, set of constraints and non-negative restrictions then is called an LPP.

Characteristics / Properties of LPP:-

i) Every linear programming problem has two important features in common. They are, an objective function to be maximized or minimized and constraints or restrictions.

ii) The graph of LPP will be a straight line.

Mathematical Formulation of LPP:-

The following steps involved in the formulation of LPP.

1. Identify the decision variables in the problem.
2. Identify the objective function which is to be maximized or minimized.
3. Identify the set constraints in the form of inequalities or equalities.
4. Identify the non-negative restrictions in the problem.

Graphical Method:- Simple LPP's of two decession variables can be solved by using graphical method.

The following steps involved in this method.

1. Convert all the inequalities in to equalities.
2. plot each equation on the graph as each one will geometrically represent a straight line.
3. Identify the feasible region. If the inequalities of the constraints are $\leq$ type then the region below the line in the first quadrant is shaded. If the in-equality of the constraints is $\geq$ type then the region above the line in the first quadrant is shaded. The region which is common to all the constraints is called feasible region.
4. The feasible region obtained in step 3 may be bounded or unbounded. Compute the co-ordinates of all the corner points of the feasible region.
5. Find out the value of the objective function at each corner point obtained in step 4.
6. select the corner point that optimizes (maximize or minimize) the value of the objective function. It gives us the optimum solution.

Exceptional Cases in Graphical Solution of LPP :-

There are some linear Programming Problems do not have solution are treat-ed as some exceptional cases in the graphical solution. Such problems occur rare-ly in real situation. There are, in general two cases arise, which can be discusse-d below.

Unbounded Solution :- If the value of objective function increases infinitely when the value of decision variables increase infinitely for a linear Programming Problem, then the solution is called an unbounded solution.

Infeasible Solution :- If there is no feasible region common to all the constrain-ts of a linear Programming Problem, then solution does not exist for such Problems, is called an infeasible solution.

Slack variable :- The non-negative variable which is added to the L.H.S of the constraint to make it as equality type is called slack variable. Slack variables are used only for $\leq$ type constraints.

Surplus variable :- The non-negative variable which is subtracted from the L.H.S of the constraint to make it as equality type is called surplus variable. Surplus variables are used only for $\geq$ type constraints.

Ex: $3x_1 + 5x_2 \geq 15$ is inequation, then S_2 is surplus variable, it should be added so

$$3x_1 + 5x_2 = 15 + S_2 \Rightarrow 3x_1 + 5x_2 - S_2 = 15$$

where, $x_1, x_2, S_2 \geq 0$

Artificial variable :- In order to obtain an IBFS, we first put the given LPP into its standard form and then a non-negative variable is added to the L.H.S of each of equation that lacks the much needed starting basic variables. The so-added variable is called artificial variable. Artifical variables are used for $\geq$ or $=$ type constraints.

Standard form of an LPP:- The standard form of an LPP is given by

$$\text{Max } Z = c_1 x_1 + c_2 x_2 + \cdots + c_n x_n$$

(or)

$$\text{Min } Z$$

Subject to the constraints

$$a_{11} x_1 + a_{12} x_2 + \cdots + a_{1n} x_n = b_1$$

$$a_{21} x_1 + a_{22} x_2 + \cdots + a_{2n} x_n = b_2$$

$$\vdots$$

$$a_{m1} x_1 + a_{m2} x_2 + \cdots + a_{mn} x_n = b_m$$

Characteristics:-

1. objective function is maximization type or minimization type.
2. All the constraints are equality type.
3. Decision variables are non-negative.

Canonical form of an LPP:- The Canonical form of an LPP is given by

$$\text{Max } Z = c_1 x_1 + c_2 x_2 + \cdots + c_n x_n$$

Subject to the constraints

$$a_{11} x_1 + a_{12} x_2 + \cdots + a_{1n} x_n \le b_1$$

$$a_{21} x_1 + a_{22} x_2 + \cdots + a_{2n} x_n \le b_2$$

$$\vdots$$

$$a_{m1} x_1 + a_{m2} x_2 + \cdots + a_{mn} x_n \le b_m$$

and $x_j \ge 0 \quad j = 1, 2 \ldots n$

Characteristics :-

1. objective function is maximization type.
2. All the constraints are $\leq$ type.
3. Decision variables are non-negative.

Matrix form of an LPP :- The matrix form of an LPP is given by

$$\text{Max } Z = Cx$$

Subject to the contraints $Ax = B$

and $x \geq 0$

where $A = [a_{ij}]_{m \times n}$ $\quad x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$ $\quad B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$

$$C = [c_1 c_2 \ldots c_n]$$

characteristics :-

1. objective function is maximization type.

Simplex Method :- Simplex method is an iterative Procedure to solve a linear Programming Problem in a finite number of steps so that the value of the objective function is improved. This Procedure is applied until an optimum basic feasible Solution is attained. The computational Procedure of simplex method is explained below.

Simplex Method :- The following steps involved in this method.

Step1 :- check whether the objective function of the given LPP is to be maximized or minimized. If it is minimized then convert into maximum by using

$$\text{Minimum } Z = -\text{Maximum } Z^*$$

where $Z^* = -Z$

Step 2 :- Check whether all $b_i (i = 1, 2 \ldots m)$ are non-negative. If any one of b_i is negative then multiply the corresponding inequation of the constraint by '-1' so as to get all b_i's $(i = 1, 2 \ldots m)$ are non-negative.

Step 3 :- Convert all the in-equality constraints into equality by introducing slack and/or surplus variables the constraints. Put the costs of these variables equal to zero in the objective function.

Step 4 :- Find an IBFS to the problem in the form $X_B = B^{-1} b$ and put it in the first column of the simplex table.

Step 5 :- Compute the net-evaluations $z_j - C_j$ $(j = 1, 2 \ldots n)$ by using the relation

$$z_j - C_j = C_B Y_j - C_j$$

Examine the sign of $z_j - C_j$.

i) If all $z_j - C_j \geq 0$ then we have been reached the optimum solution.

ii) If atleast one $z_j - C_j < 0$ then go to next step.

Step 6 :- If there are more than one negative $z_j - C_j$ then choose the most negative of them. let it be

i) If all $Y_{ir} \leq 0$ $(i = 1, 2 \ldots m)$ then there is an unbounded solution to the given problem.

ii) If atleast one $Y_{ir} > 0$ $(i = 1, 2 \ldots m)$ then the corresponding vector Y_n enters the basis Y_B.

Step 7 :- Compute the ratios $\left\{ \frac{X_{Bi}}{Y_{ir}}, Y_{ir} > 0 \quad i = 1, 2 \ldots m \right\}$ and choose the minimum of them. Let the minimum of these ratios be X_{BK}/Y_{Kn}. Then the vector Y_K will leave the basis Y_B. The common element Y_{Kr}, which is in the Kth row and r^{th} column is known as leading or key or pivotal element.

Step 8 :- Convert the leading element to unity by dividing itself and all other elements in its column is equal to zero by performing matrix row operations.

Step 9 :- Go to Step 5 and repeat the computational procedure until either an optimum solution is reached or there is an indication of an unbounded solution.

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Problems on Formulation of LPP :-

18. A company manufactures two type of Products A and B and sells them at a profit of RS. 2 on type A and RS. 3 on type B. Each Product is processed on two machines. Product A requires one minute of processing time on machine I and two minutes on machine II, Product B requires two minutes on machine I and three minutes on machine II. The machine I is available for not more than 6 hours while machine II is available for 10 hours during any working day. Formulate the Problem as a linear Programming Problem.

Sol. The Given Problem can be expressed in a tabular form as follows.

Products	Machine M_1	Machine M_2	Profit
Product $A(x_1)$	1	2	2
Product $B(x_2)$	2	3	3
Availability	$=6\times60$ 360	$=10\times60$ 600	

i) Decision variables :- let x_1 & x_2 be the decision variables.

where x_1 represents no. of Products of type A

x_2 represents no. of Products of type B

ii) objective function :- Max $Z = 2x_1 + 3x_2$

iii) Set of Constraints :- $x_1 + 2x_2 \leq 360$ (Machine M_1)

$2x_1 + 3x_2 \leq 600$ (Machine M_2)

iv) Non-Negative restrictions :- $x_1, x_2 \geq 0$

∴ The LPP to the given Problem as follows.

Max $Z = 2x_1 + 3x_2$

S.O.C :- $x_1 + 2x_2 \leq 360$ (Machine M_1)

$2x_1 + 3x_2 \leq 600$ (Machine M_2)

and $x_1, x_2 \geq 0$ //

RQ. A firm manufactures headache pills in two sizes A and B. Size A contains 2 grains of aspirin, 5 grains of bicarbonate and 1 grain of codeine. Size B contains 1 grain of aspirin, 8 grains of bicarbonate and 6 grains of codeine. It is found by users that it requires at least 12 grains of aspirin, 74 grains of bicarbonate and 24 grains of codeine for providing immediate effect. Formulate the problem as a standard LPP.

Sol. The Given Problem can be expressed in a tabular form as follows.

Pills	Aspirin	Bicarbonate	Codeine	Profit
Size A	2	5	1	1
Size B	1	8	6	1
Availability	12	74	24	

i) Decision variables:- let x_1 & x_2 be the decision variables

where x_1 represents the no. of pills of size A.

x_2 represents the no. of pills of size B.

ii) objective function:- $\text{Max } Z = x_1 + x_2$

iii) Set of Constraints:- $2x_1 + x_2 \geq 12$ (Aspirin)

$5x_1 + 8x_2 \geq 74$ (Bicarbonate)

$x_1 + 6x_2 \geq 24$ (codeine)

iv) Non-negative Restrictions:- x_1 & $x_2 \geq 0$

∴ The LPP to the given problem as follows.

$\text{Max } Z = x_1 + x_2$

SOC = $2x_1 + x_2 \geq 12$ (Aspirin)

$5x_1 + 8x_2 \geq 74$ (Bicarbonate)

$x_1 + 6x_2 \geq 24$ (codeine)

and $x_1, x_2 \geq 0$

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55. The manager of an oil refinery must decide on the optimum mix of 2 possible blending process of which the ~~inputs~~ inputs and outputs production run are as follows:

Process	Input		Output	
	Crude A	Crude B	Gasoline X	Gasoline Y
1	6	4	6	9
2	5	6	5	5

The maximum amounts available of Crudes A and B are 250 units and 200 units respectively. Market demand shows that at least 150 units of gasoline X and 130 units of gasoline Y must be produced. The profits per production run from Process 1 and Process 2 are Rs. 4 and Rs. 5 respectively. Formulate the problem for maximising the profit.

Sol. The Given Problem can be expressed in a tabular form as follows.

Process	Input		output		Profit.
	Crude A	Crude B	Gasoline X	Gasoline Y	
1	6	4	6	9	4
2	5	6	5	5	5
Availability	250	200	150	130	

i) Decision variables :- let x_1 & x_2 be the decision variables

where x_1 represents the Process A.

x_2 represents the Process B.

ii) objective function :- $\text{Max } Z = 4x_1 + 5x_2$

iii) Set of constraints :- $6x_1 + 5x_2 \leq 250$ (crude A)

$4x_1 + 6x_2 \leq 200$ (Crude B)

$6x_1 + 5x_2 \geq 150$ (Gasoline X)

$9x_1 + 5x_2 \geq 130$ (Gasoline Y)

iv) Non-negative Restrictions :- $x_1, x_2 \geq 0$

∴ The LPP to the given problem as follows

$$\text{Max } Z = 4x_1 + 5x_2$$

$$\text{SOC} = 6x_1 + 5x_2 \leq 250 \text{ (crude A)}$$
$$4x_1 + 6x_2 \leq 200 \text{ (crude B)}$$
$$6x_1 + 5x_2 \geq 150 \text{ (Gasoline X)}$$
$$9x_1 + 5x_2 \geq 130 \text{ (Gasoline Y)}$$

and $x_1, x_2 \geq 0$ //

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Problems on Graphical Method :-

1Q. Solve the following LPP by using graphically. (Alternative optimum solution)

$$\text{Max } Z = 2x_1 + 3x_2$$

Set of constraints : $x_1 + 2x_2 \le 360$

$2x_1 + 3x_2 \le 600$

and $x_1, x_2 \ge 0$

Sol: i) Convert all the inequality constraints into equality constraints, then we get

$x_1 + 2x_2 = 360 \rightarrow (1)$

$2x_1 + 3x_2 = 600 \rightarrow (2)$

ii) plot each eq. of the graph as each one will geometrically represent a straight line.

From (1)

Put $x_1 = 0 \Rightarrow 2x_2 = 360 \Rightarrow x_2 = 180$

Put $x_2 = 0 \Rightarrow x_1 = 360$

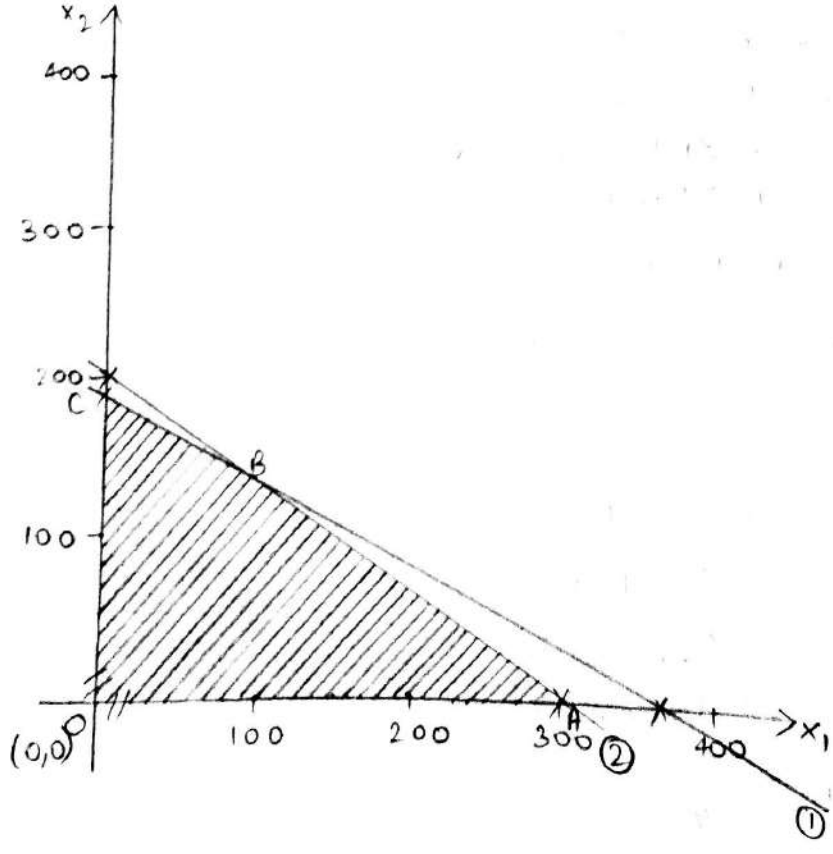
$\Big\}$ (360, 180)

From (2)

Put $x_1 = 0 \Rightarrow 3x_2 = 600 \Rightarrow x_2 = 200$

Put $x_2 = 0 \Rightarrow 2x_1 = 600 \Rightarrow x_1 = 300$

$\Big\}$ (300, 200)



iii) OABC is the feasible region

iv) Identify the corner points of the feasible region

$O = (0,0)$

$A = (a,0) = (300,0)$

$B =$ Intersection of ① and ②

$① \times 2 = 2x_1 + 4x_2 = 720$

use ② $= 2x_1 + 3x_2 = 600$

$x_2 = 120$

from ①

$x_1 + 2(120) = 360$

$x_1 = 360 - 240$

$x_1 = 120$

$C = (0,a) = (0,180)$

v) Given that $\text{Max } Z = 2x_1 + 3x_2$

$Z_O = 2(0) + 3(0) = 0$

$Z_A = 2(300) + 3(0) = 600$

$Z_B = 2(120) + 3(120) = 240 + 360 = 600$

$Z_C = 2(0) + 3(180) = 540$

vi) The optimum value to the given LPP is

Max $Z = 600$ at the points $A(300,0)$ & $B(120,120)$

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RQ. Solve the following LPP graphically.

$$\text{Max } Z = 45x_1 + 80x_2$$

Set of constraints :

$$5x_1 + 20x_2 \le 400$$

$$10x_1 + 15x_2 \le 450$$

and $x_1, x_2 \ge 0$.

Sol. i) Convert all the inequality constraints into equality constraints, then we get

$\cancel{x_1 + 2x_2 = 360}$ $5x_1 + 20x_2 = 400 \rightarrow (1)$

$10x_1 + 15x_2 = 450 \rightarrow (2)$

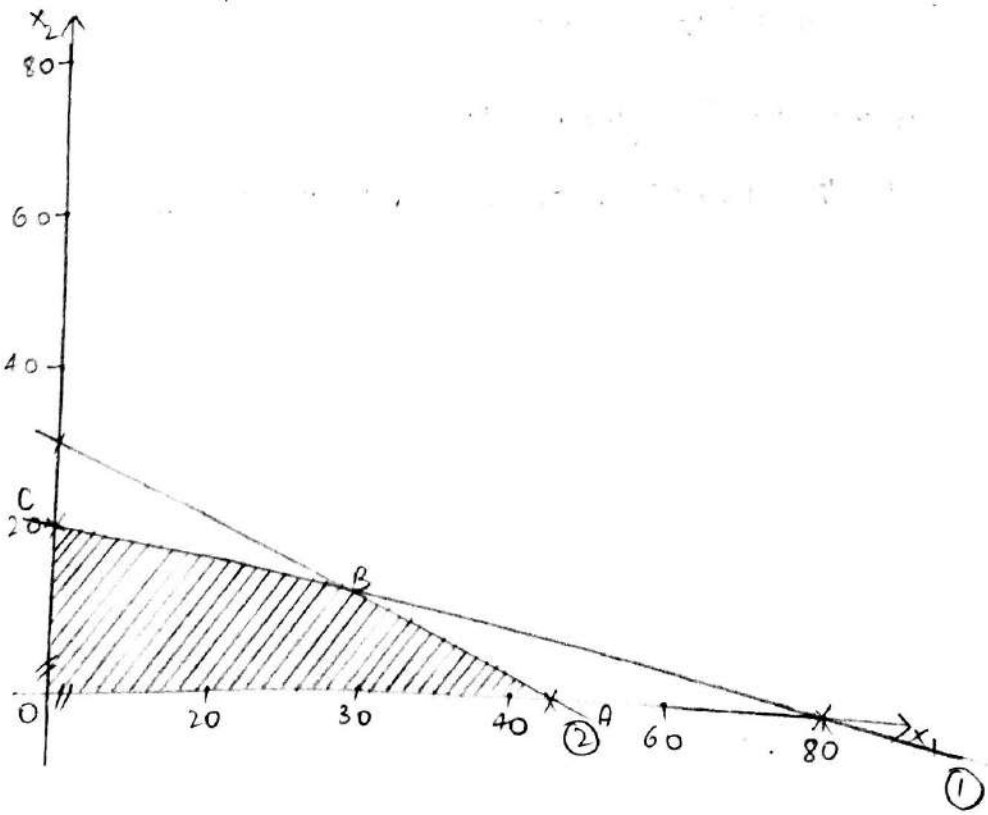
ii) plot each eq. of the graph as each one will geometrically represent a straight line.

From (1)

Put $x_1 = 0 \Rightarrow 20x_2 = 400 \Rightarrow x_2 = 20$
Put $x_2 = 0 \Rightarrow 5x_1 = 400 \Rightarrow x_1 = 80$ } $(80, 20)$

From (2)

Put $x_1 = 0 \Rightarrow 15x_2 = 450 \Rightarrow x_2 = 30$
Put $x_2 = 0 \Rightarrow 10x_1 = 450 \Rightarrow x_1 = 45$ } $(45, 30)$



iii) OABC is the feasible region

iv) Identify the corner points of the feasible region.

$O = (0,0)$

$A = (a,0) = (450,0)$

B = Intersection of ① and ②

① × 2 $= \cancel{10x_1} + 40x_2 = 800$

② $= \cancel{10x_1} + 15x_2 = 450$

$25x_2 = 350$

$x_2 = 14$

we get :

from ①

$5x_1 + 20(14) = 400$

$5x_1 = 400 - 280$

$5x_1 = 120$

$x_1 = 24$

aight

$B = (24,14)$

$C = (0,a) = (0,20)$

v) Given that $\text{Max } Z = 45x_1 + 80x_2$

$Z_O = 45(0) + 80(0) = 0$

~~$Z_A = 45(24) + 80(14) = 2200$~~

~~$Z_B = 45$~~

$Z_A = 45(45) + 80(0) = 2025$

$Z_B = 45(24) + 80(14) = 2200$

$Z_C = 45(0) + 80(20) = 1600$

vi) The optimum value to the given LPP is

$\text{Max } Z = 2200$ at the points $B(24,14)$

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3Q. Solve the following LPP graphically.

$$\text{Min } Z = 4x_1 + 2x_2$$

Subject to the constraints :- $x_1 + 2x_2 \geq 2$

$3x_1 + x_2 \geq 6\ 3$

$4x_1 + 3x_2 \geq 6$

and $x_1, x_2 \geq 0$

Sol. i) convert all the inequality constraints into equality constraints, then we get

$x_1 + 2x_2 = 2 \rightarrow (1)$

$3x_1 + x_2 = 3 \rightarrow (2)$

$4x_1 + 3x_2 = 6 \rightarrow (3)$

ii) Plot each equation of the graph as each one will geometrically represent a straight line.

from (1)

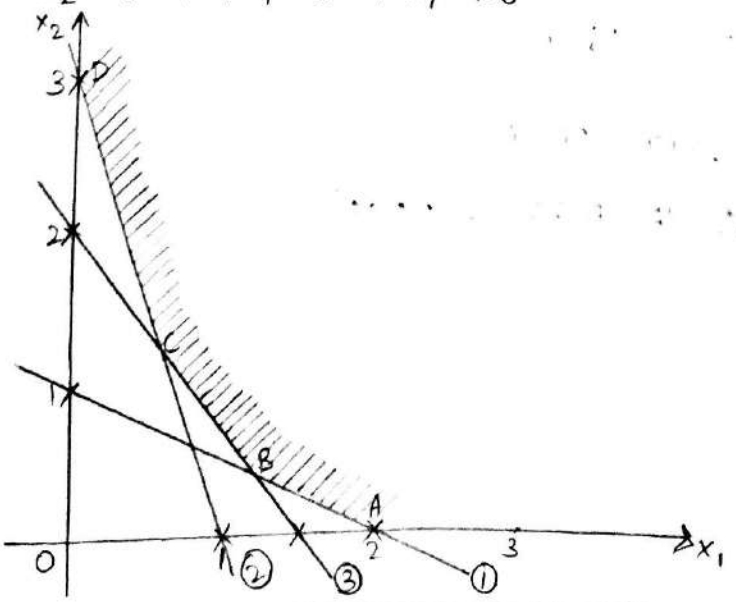
Put $x_1 = 0 \Rightarrow 2x_2 = 2 \Rightarrow x_2 = 1$
Put $x_2 = 0 \Rightarrow x_1 = 2$ } $(2,1)$

from (2)

Put $x_1 = 0 \Rightarrow x_2 = 3$
Put $x_2 = 0 \Rightarrow 3x_1 = 3 \Rightarrow x_1 = 1$ } $(1,3)$

from (3)

Put $x_1 = 0 \Rightarrow 3x_2 = 6 \Rightarrow x_2 = 2$
Put $x_2 = 0 \Rightarrow 4x_1 = 6 \Rightarrow x_1 = 1.5$ } $(1.5, 2)$



iii) ABCD is the feasible region.

iv) Identify the corner points of the feasible region.

$A = (a, 0) = (2, 0)$

B = Intersection of ① and ③

$① \times 4 = \cancel{4x_1} + 8x_2 = 8$

$③ \quad = \cancel{4x_1} + 3x_2 = 6$

$5x_2 = 2$

$x_2 = 0.4$

from ①

$x_1 + 2(0.4) = 2$

$x_1 = 2 - 0.8$

$x_1 = 1.2$

$B = (1.2, 0.4)$

C = Intersection of ② & ③

$② \times 3 = 9x_1 + \cancel{3x_2} = 9$

$③ \quad = 4x_1 + \cancel{3x_2} = 6$

$5x_1 = 3$

$x_1 = 3/5$

$x_1 = 0.6$

from ② $3(0.6) + x_2 = 3$

$x_2 = 3 - 1.8$

$x_2 = 1.2$

$C = (0.6, 1.2)$

$D = (0, a) = (0, 3)$

v) Given that Min $Z = 4x_1 + 2x_2$

$Z_A = 4(2) + 2(0) = 8$

$Z_B = 4(1.2) + 2(0.4) = 5.6$

$Z_C = 4(0.6) + 2(1.2) = 4.8$

$Z_D = 4(0) + 2(3) = 6$

vi) The optimum value to the given LPP is Min $Z = 4.8$ at the points $c(0.6, 1.$

4Q. Solve the following LPP graphically.

$$\text{Min } Z = 1.5x_1 + 2.5x_2$$

Subject to the constraints - $x_1 + 3x_2 \geq 3$

$x_1 + x_2 \geq 2$

and $x_1, x_2 \geq 0$

Sol. i) Convert all the inequality constraints into equality constraints, then we get

$x_1 + 3x_2 = 3 \rightarrow (1)$

$x_1 + x_2 = 2 \rightarrow (2)$

ii) Plot each equation of the graph as each one will geometrically represent a straight line

from (1)

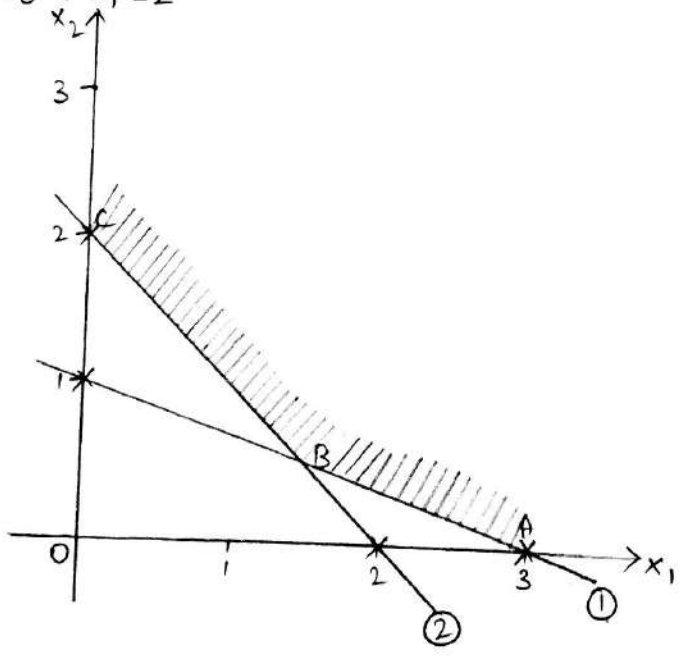
Put $x_1 = 0 \Rightarrow 3x_2 = 3 \Rightarrow x_2 = 1$

Put $x_2 = 0 \Rightarrow x_1 = 3$ } $(3, 1)$

from (2)

Put $x_1 = 0 \Rightarrow x_2 = 2$

Put $x_2 = 0 \Rightarrow x_1 = 2$ } $(2, 2)$



iii) ABC is the feasible region.

iv) Identify the corner points of the feasible region

$A = (a, 0) = (3, 0)$

B = Intersection of ① & ②

$$x_1 + 3x_2 = 3$$
$$\underline{x_1 + x_2 = 2}$$
$$2x_2 = 1$$
$$x_2 = 0.5$$

get from ②

$$x_1 + 0.5 = 2$$
$$x_1 = 2 - 0.5$$
$$x_1 = 1.5$$
$$B = (1.5, 0.5)$$

$$C = (0, a) = (0, 2)$$

Given that $\text{Min } Z = 1.5x_1 + 2.5x_2$.

$$Z_A = 1.5(3) + 2.5(0)$$
$$Z_A = 4.5$$
$$Z_B = 1.5(1.5) + 2.5(0.5)$$
$$Z_B = 3.5$$
$$Z_C = 1.5(0) + 2.5(2) = 5$$

The optimum value to the given LPP is

$\text{Min } Z = 3.5$ at the points $B(1.5, 0.5)$

5Q. Solve the following LPP graphically

$$\text{Max } Z = 120x_1 + 100x_2$$

Subject to the constraints = $10x_1 + 5x_2 \leq 80$

$$6x_1 + 6x_2 \leq 66$$

$$4x_1 + 8x_2 \geq 24$$

$$5x_1 + 6x_2 \leq 90$$

and $x_1, x_2 \geq 0$

Sol. i) Convert all the inequality constraints into equality constraints then we get

$10x_1 + 5x_2 = 80 \rightarrow (1)$

$6x_1 + 6x_2 = 66 \Rightarrow x_1 + x_2 = 11 \rightarrow (2)$

$4x_1 + 8x_2 = 24 \rightarrow (3)$

$5x_1 + 6x_2 = 90 \rightarrow (4)$

ii) plot each equation of the graph as each one will geometrically represent a straight line

from (1)

$$\left.\begin{array}{l} \text{Put } x_1 = 0 \Rightarrow 5x_2 = 80 \Rightarrow x_2 = 16 \\ \text{Put } x_2 = 0 \Rightarrow 10x_1 = 80 \Rightarrow x_1 = 8 \end{array}\right\} (8, 16)$$

from (2)

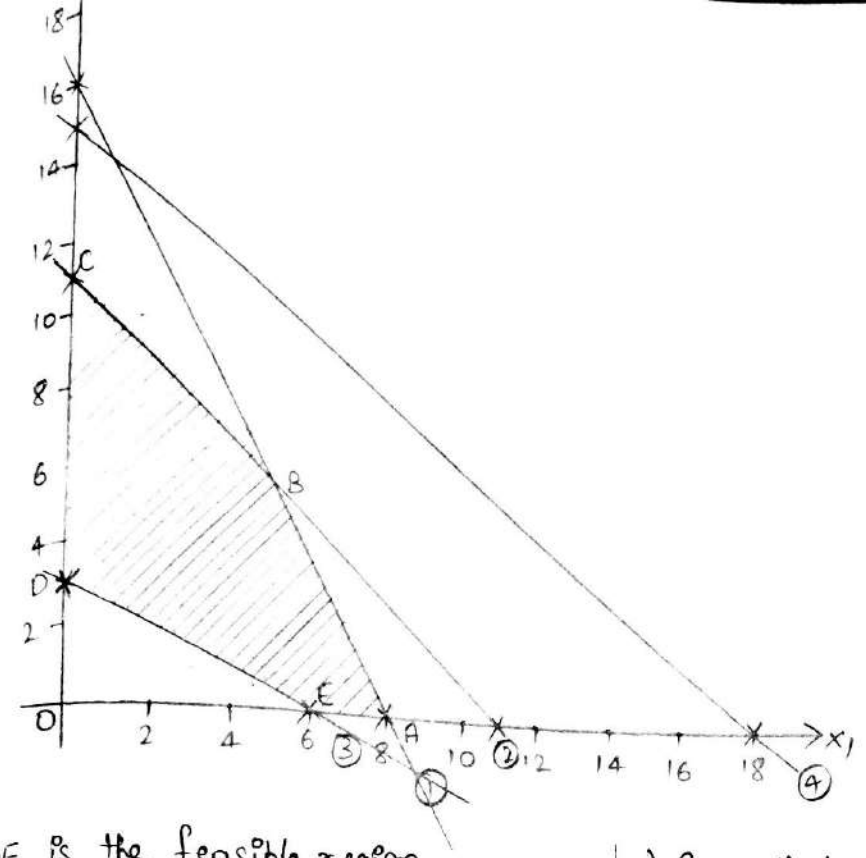
$$\left.\begin{array}{l} \text{Put } x_1 = 0 \Rightarrow x_2 = 11 \\ \text{Put } x_2 = 0 \Rightarrow x_1 = 11 \end{array}\right\} (11, 11)$$

from (3)

$$\left.\begin{array}{l} \text{Put } x_1 = 0 \Rightarrow 8x_2 = 24 \Rightarrow x_2 = 3 \\ \text{Put } x_2 = 0 \Rightarrow 4x_1 = 24 \Rightarrow x_1 = 6 \end{array}\right\} (6, 3)$$

from (4)

$$\left.\begin{array}{l} \text{Put } x_1 = 0 \Rightarrow 6x_2 = 90 \Rightarrow x_2 = 15 \\ \text{Put } x_2 = 0 \Rightarrow 5x_1 = 90 \Rightarrow x_1 = 18 \end{array}\right\} (18, 15)$$



∴ ABCDE is the feasible region.

Identify the Corner Points of the feasible region.

$A = (a, 0) = (8, 0)$

B = Intersection of ① & ②

① = $\cancel{10x_1} + 5x_2 = 80$

$3 \times 10 = \cancel{10x_1} + 10x_2 = 110$

$-5x_2 = -30$

$x_2 = 6$

from ②

$x_1 + 6 = 11$

$x_1 = 5$

$B = (5, 6)$

$C = (0, a) = (0, 11)$

$D = (0, a) = (0, 3)$

$E = (a, 0) = (6, 0)$

v) Given that $\text{Max } Z = 120x_1 + 100x_2$

$Z_A = 120(8) + 100(0) = 960$

$Z_B = 120(5) + 100(6) = 1200$

$Z_C = 120(0) + 100(11) = 1100$

$Z_D = 120(0) + 100(3) = 300$

$Z_E = 120(6) + 100(0) = 720$

vi) The optimum value to the given LPP is

$\text{Max } Z = 1200$ at the point $B(5, 6)$.

19/5/22

Problem on Unbounded Solution :-

1Q. Solve the following LPP by using graphical method.

$$\text{Max } Z = 3x_1 + 2x_2$$

Subject of constraints :- $-2x_1 + 3x_2 \leq 9$

$$x_1 - 5x_2 \geq -20$$

and $x_1, x_2 \geq 0$

Sol. Convert all the inequality constraints into equality constraints, then we get

$$-2x_1 + 3x_2 = 9 \rightarrow (1)$$

$$x_1 - 5x_2 = -20 \rightarrow (2)$$

Plot each equation of the graph as each one will geometrically represent a straight line

from (1)

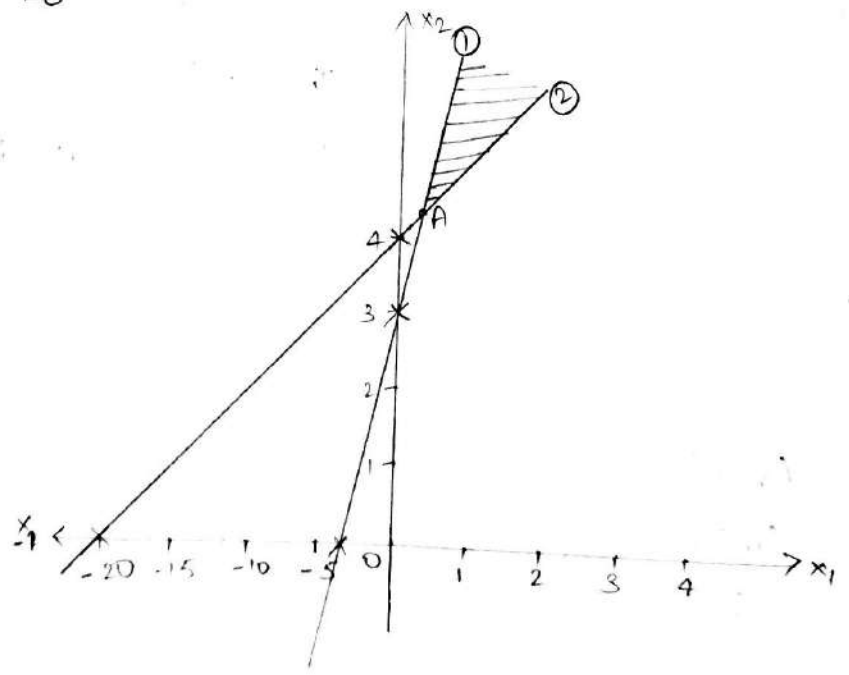
Put $x_1 = 0 \Rightarrow 3x_2 = 9 \Rightarrow x_2 = 3$

Put $x_2 = 0 \Rightarrow -2x_1 = 9 \Rightarrow x_1 = \frac{-9}{2} = -4.5$ $\Big\}$ $(-4.5, 3)$

from (2)

Put $x_1 = 0 \Rightarrow -5x_2 = -20 \Rightarrow x_2 = 4$

Put $x_2 = 0 \Rightarrow x_1 = -20$ $\Big\}$ $(-20, 4)$



A = Intersection of (1) & (2)

① $= -2x_1 + 3x_2 = 9$

②$\times 2 = 2x_1 - 10x_2 = -40$

$-7x_2 = -31$

$x_2 = 4.4$

from ①

$x_1 - 5(4.4) = -20$

$x_1 = -20 + 22$

$x_1 = 2$

$A = (2, 4.4)$

Given that $\text{Max } Z = 3x_1 + 2x_2$

$Z_A = 3(2) + 2(4.4)$

$Z_A = 6 + 8.8$

$Z_A = 14.8$

This is an unbounded solution because there are so many points exists which optimises the objective function more than 14.8. ~~for e~~

For example: Consider a point (8, 20) exists in the feasible region.

$\therefore \text{Max } Z = 3(8) + 2(20)$

$= 24 + 40$

$\text{Max } Z = 48 > 14.8$ //

Problem on Infeasible Solution :-

1Q. Solve the following LPP by using Graphical method.

$$\text{Max } Z = 3x_1 - 2x_2$$

Subject of Constraints :- $x_1 + x_2 \leq 1$

$x_1 + x_2 \geq 2$

and $x_1, x_2 \geq 0$

Sol. Convert all the inequality constraints into equality constraints then we get

$x_1 + x_2 = 1 \rightarrow (1)$

$x_1 + x_2 = 2 \rightarrow (2)$

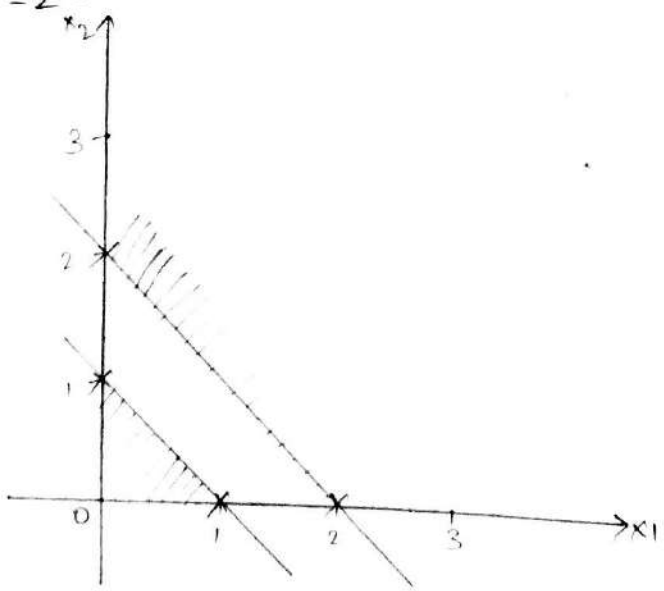
Plot each equation of the graph as each one will geometrically represent straight line

from (1)

$$\left.\begin{array}{l}\text{Put } x_1 = 0 \Rightarrow x_2 = 1 \\ \text{Put } x_2 = 0 \Rightarrow x_1 = 1\end{array}\right\} (1,1)$$

from (2)

$$\left.\begin{array}{l}\text{Put } x_1 = 0 \rightarrow x_2 = 2 \\ \text{Put } x_2 = 0 \Rightarrow x_1 = 2\end{array}\right\} (2,2)$$



∴ There exists no common region to the given LPP.

∴ The LPP has an infeasible solution.

20/5/R.R.

Problems on simplex Method:-

18. Solve the following LPP.

$$\text{Max } Z = 4x_1 + 10x_2$$

Subject of the constraints :- $2x_1 + x_2 \le 50$

$$2x_1 + 5x_2 \le 100$$

$$2x_1 + 3x_2 \le 90$$

and $x_1, x_2 \ge 0$

Sol. The given LPP is maximization and all b_i's (right side constants) are non-negative. By converting given inequations of the constraints into equations.

$$2x_1 + x_2 = 50$$

$$2x_1 + 5x_2 = 100$$

$$2x_1 + 3x_2 = 90$$

Introduce a slack variable $x_3, x_4, x_5 \ge 0$ and put the cost of three variables $= 0$ in the object function, then we get

$$\text{Max } Z = 4x_1 + 10x_2 + 0x_3 + 0x_4 + 0x_5$$

Subject of the constraints :- $2x_1 + x_2 + x_3 + 0x_4 + 0x_5 = 50$

$$2x_1 + 5x_2 + 0x_3 + x_4 + 0x_5 = 100$$

$$2x_1 + 3x_2 + 0x_3 + 0x_4 + x_5 = 90$$

and $x_j \ge 0 \; j = 1, 2, 3, 4, 5$

Matrix form :-

$$Ax = B$$

$$A = \begin{bmatrix} 2 & 1 & 1 & 0 & 0 \\ 2 & 5 & 0 & 1 & 0 \\ 2 & 3 & 0 & 0 & 1 \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} \quad B = \begin{bmatrix} 50 \\ 100 \\ 90 \end{bmatrix} \quad C = \begin{bmatrix} 4 & 10 & 0 & 0 & 0 \end{bmatrix}$$

P.T.O

Starting table:

		C_j	4	10	0	0	0
Y_B	C_B	X_B	Y_1	Y_2	Y_3	Y_4	Y_5
Y_3	0	50	2	1	1	0	0
Y_4	0	100	2	$\boxed{5}$	0	1	0
Y_5	0	90	2	3	0	0	1
$Z = 0$		$Z_j - C_j$	-4	-10	0	0	0

↑

Ratio

Min $\{X_B/Y_2\}$

$\frac{50}{1} = 50$ $\quad R_1 \to R_1 - \frac{1}{5}R_2$

$\frac{100}{5} = 20 \to R_2 \to \frac{R_2}{5}$

$\frac{90}{3} = 30$ $\quad R_3 \to R_3 - \frac{3R_2}{5}$

$Z = \Sigma C_B X_B$

$Z_j - C_j = \Sigma C_B Y_j - C_j$

$Z_2 - C_2$ is the most negative (-10).

$\therefore Y_2$ enters into the basis.

Y_4 is the least positive (20). so it leaves the bases.

Identify the key element (or) leading element (or) pipotal element (the element which is common to entering columns and deleting row) = 5

$R_1 \longrightarrow R_1 - \frac{R_2}{5}$

$R_2 \longrightarrow \frac{R_2}{5}$

$R_3 \longrightarrow R_3 - \frac{3R_2}{5}$

$$R_1 - \frac{R_2}{5} = \begin{array}{cccccc} 50 & 2 & 1 & 1 & 0 & 0 \\ - & - & - & - & - & - \\ 20 & 2/5 & 1 & 0 & 1/5 & 0 \\ \hline 30 & 8/5 & 0 & 1 & -1/5 & 0 \end{array}$$

$$R_3 - \frac{3R_2}{5} = \begin{array}{cccccc} 90 & 2 & 3 & 0 & 0 & 1 \\ - & - & - & - & - & - \\ 60 & 6/5 & 3 & 0 & 3/5 & 0 \\ \hline 30 & 4/5 & 0 & 0 & -3/5 & 1 \end{array}$$

Second table:

		C_j	4	10	0	0	0
Y_B	C_B	X_B	Y_1	Y_2	Y_3	Y_4	Y_5
Y_3	0	30	8/5	0	1	-1/5	0
Y_2	10	20	2/5	1	0	1/5	0
Y_5	0	30	4/5	0	0	-3/5	1
$Z = 200$		$Z_j - C_j$	0	0	0	2	0

Max $Z = 200$

for the variables $x_1 = 0$, $x_2 = 20$

Max $Z = 4x_1 + 10x_2$

21/5/22 RQ. Solve the following LPP by using simplex method.

Max $Z = 3x_1 + 2x_2$ (Unbounded Solution)

subject of the constraints :- $-2x_1 + 3x_2 \le 9$

$x_1 - 5x_2 \ge -20$

and $x_1, x_2 \ge 0$

Sol. ~~Since all bi's~~ The given LPP is maximization Problem.

on of the bi's (right side constant) is negative so multiply the constraint with '-1' then constraints are :

$$-2x_1 + 3x_2 \le 9$$

$$-x_1 + 5x_2 \le 20$$

and $x_1, x_2 \ge 0$

By converting inequations of constraints into equations and introduce the slack variables $x_3, x_4 \ge 0$ and put the cost of these variables $= 0$ in the object function, then we get

Max $Z = 3x_1 + 2x_2 + 0x_3 + 0x_4$

Subject of the constraints :- $-2x_1 + 3x_2 + x_3 + 0x_4 = 9$

$-x_1 + 5x_2 + 0x_3 + x_4 = 20$

and $x_j \ge 0$ $j = 1, 2, 3, 4$

Matrix form :-

$Ax = B$

$$A = \begin{bmatrix} -2 & 3 & 1 & 0 \\ -1 & 5 & 0 & 1 \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$B = \begin{bmatrix} 9 \\ 20 \end{bmatrix} \quad C = \begin{bmatrix} 3 & 2 & 0 & 0 \end{bmatrix}$$

Starting table :-

		C_j	3	2	0	0	
Y_B	C_B	X_B	Y_1	Y_2	Y_3	Y_4	Ratio
Y_3	0	9	-2	3	1	0	Min$\{x_B/y_1\}$
Y_4	0	20	-1	5	0	1	
$Z = 0$		$Z_j - C_j$	-3	-2	0	0	

$Z_1 - C_1$ is the most negative (-3). ∴ Y_1 enters into the basis. But all the elements in Y_1 are ≤ 0 (-2, -1).

∴ Deleting variable doesnot exists. ∴ The Given LPP has an unbounded Solution.

3Q. Solve the following LPP by using simplex Method.

$$\text{Min } Z = 8x_1 - 2x_2$$

Subject of the Constraints :- $-4x_1 + 2x_2 \le 1$

$$5x_1 - 4x_2 \le 3$$

and $x_1, x_2 \ge 0$.

Sol. The given LPP is of minimization type.

Hence we need to convert into Maximization.

$$\text{Max } Z^* = -\text{Min } Z$$

$$\text{Max } Z^* = -[8x_1 - 2x_2]$$

$$\text{Max } Z^* = -8x_1 + 2x_2$$

Introduce the slack variables $x_3, x_4 \ge 0$ and Put the cost of these variables $=0$ in the object function then we get.

$$\text{Ma } Z^* = -8x_1 + 2x_2 + 0x_3 + 0x_4$$

$$\text{SOC :- } -4x_1 + 2x_2 + x_3 + 0x_4 = 1$$

$$5x_1 - 4x_2 + 0x_3 + x_4 = 3$$

and $x_j \ge 0$, $j = 1, 2, 3, 4$

Matrix form :-

$$Ax = B$$

$$A = \begin{bmatrix} -4 & 2 & 1 & 0 \\ 5 & -4 & 0 & 1 \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \quad C = \begin{bmatrix} -8 & 2 & 0 & 0 \end{bmatrix}$$

Starting table :-

		C_j	-8	2	0	0	
Y_B	C_B	X_B	Y_1	Y_2	Y_3	Y_4	Ratio
Y_3	0	1	-4	$\boxed{2}$	1	0	$\text{Min}\{x_B/y_2\}$ $\frac{1}{2} = 0.5$
Y_4	0	3	5	-4	0	1	
$Z^* = 0$		$Z_j - C_j = 8$	8	-2	0	0	

$R_1 \to \frac{R_1}{2}$

$R_2 \to 4R_2 + \frac{\cancel{4}^{2} R_1}{\cancel{2}}$

Second table :-

		C_j	-8	0	0	0	
Y_B	C_B	X_B	Y_1	Y_2	Y_3	Y_4	Ratio
Y_2	2	$1/2$	-2	1	$1/2$	0	Min $\{*/1/2\}$
Y_4	0	5	-3	0	2	1	
$Z^* = 1$		$Z_j - C_j$	4	0	1	0	

$4R_2 + 2R_1 =$

3	5	-4	0	1
2	-8	4	2	0
5	-3	0	2	1

Since $Z_j - C_j \geq 0$. ~~then~~ Hence we have been reached the optimum solution and the optimum value of $Z = -Z^*$

$$Z = -1 \text{ for the variables}$$

$$x_1 = 0 \; , \; x_2 = 1/2$$

Operation Research :- The term 'operation Research' refers to the application of scientific methodology of several different disciplines to problems related to the functioning or operating of some unit business, government or institutions.

- Operation Research specialist help the managers to make better decisions.

Applications :-

Marketing

1. Product selection, timing, competitive actions.
2. Advertising media with respect to cost and time.
3. Number of salesmen, frequency of calling of account etc.
4. Effectiveness of market research.

Purchasing, Procurement and Exploration

1. Rules for buying
2. Determination of quantities and timing of purchases.
3. Equipment replacement policies.

Production

1. Calculating optimum production mix
2. Selection, location and design of the sites for the production plant.

Physical Distribution

1. Location and size of warehouses, distribution centres and retail outlets etc.
2. Distribution policy.

Personnel Management

1. Selection of suitable personnel on minimum salary.
2. Mixes of age and skills
3. Recruitment policies and assignment of jobs.

Research and Development

1. Reliability and evaluation of alternative designs.
2. Control of developed projects.
3. Determination of time and cost requirements.
4. Coordination of multiple research projects.

Scope of OR:- In recent years, OR has widely spread into various fields. Many number of problems can be solved by using O.R. techniques, hence the scope of operations Research is studied below in the various fields.

Agriculture:- The following are the main problems in agriculture due to rapid population growth in many countries.

1. optimum allocation of land to various crops in various climatic conditions.
2. optimum distribution of water for irrigation.

Finance:- The Government has to take necessary planning to develop the country economically for which the following main steps for consideration.

1. Maximising the per capita income with the available sources.
2. To improve various growth rates in accordance with available resources.
3. To maximise the profit of companies etc.

Industry:- operations research techniques are useful to the industrial sector for optimum allocation of the various limited resources like manpower, machine, material, money, time, etc. to maximise the profit and minimise cost.

Marketing :- The following Problems can be Solved through O.R. techniques in marketing field.

1. To maximise Sales of the Product.
2. To transport the Product at minimum cost.
3. The size of the Stock to meet future demand.

LIC :- O.R. Can be applied in LIC for the following problems:

1. To decide Premium rates for various modes of policies.
2. To prepare the best policies with respect to profit i.e., to maximise profit.

Personnel Management :-

1. To appoint the most suitable persons with minimum Salaries.
2. To select most efficient persons in the relevant field.

8. What are various models of OR by its structure?

Iconic models, Analogue models, Mathematical (or) Symbolic models.

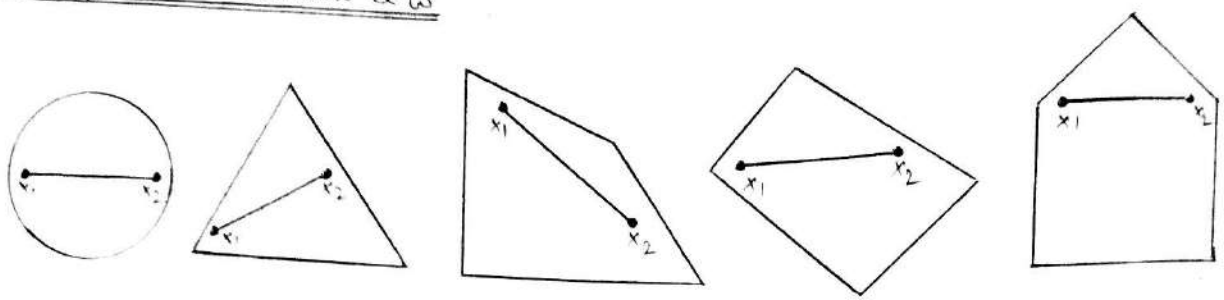
Convex Set :- A subset $S \subset \mathbb{R}^n$ is said to be convex if for any two points $x_1, x_2 \in S$, the line segment joining the points x_1 and x_2 is also contained in S.

Symbolically,

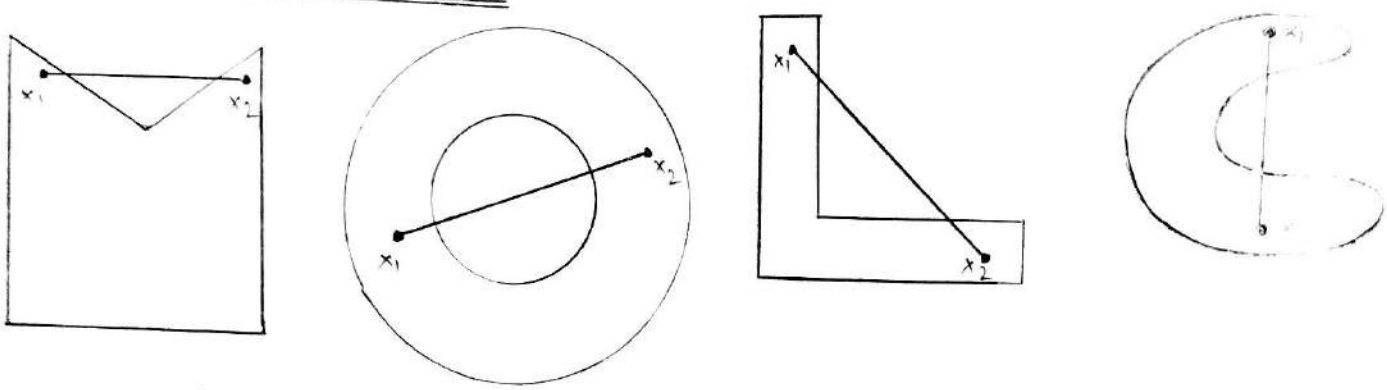
$S \subset \mathbb{R}^n$ is convex, if and only if

$x_1, x_2 \in S \Rightarrow \lambda x_1 + (1-\lambda) x_2 \in S, \; 0 \le \lambda \le 1.$

Examples of Convex Sets



Examples of non-Convex Sets



Properties of Convex Set

1. The set of all convex combinations of finite number of linearly independent vectors is a convex set.
2. The intersection of all collection of convex sets is itself a convex set.
3. The closed convex sets can be characterised as the intersections of closed half-spaces which are convex sets.

Extreme Point of a Convex Set :- A point of a convex set which does not lie on any line segment joining two other points of the convex set is called an extreme point of a convex set.

Symbolically, if x is an extreme point of a convex set, there does not exist any pair of points x_1, x_2 in S, such that

$$x = \lambda x_1 + (1-\lambda) x_2, \; 0 \le \lambda \le 1.$$

Fundamental theorem of linear Programming Problem :-

Statement :- The set of all feasible solutions of a LPP constitute a convex set.

Proof :- let us consider a LPP in matrix form to determine x so that the objective function z is maximised as follows:

$$\max Z = Cx$$

Subject to the constraints

$$Ax = b$$

and $x \geq 0$

let $x^{(1)}, x^{(2)}$ be two feasible solutions to the LPP. By the definition of feasible soln, these should satisfy the constraints and non-negative restrictions.

$$\therefore Ax^{(1)} = b, \quad Ax^{(2)} = b$$

and $x^{(1)} \geq 0, \quad x^{(2)} \geq 0$

consider x as convex linear combination of $x^{(1)}$ and $x^{(2)}$ as follows:

$$x = \lambda x^{(1)} + (1-\lambda)x^{(2)}; \; 0 \leq \lambda \leq 1$$

Consider $Ax = A[\lambda x^{(1)} + (1-\lambda)x^{(2)}]$

$$= \lambda Ax^{(1)} + (1-\lambda)Ax^{(2)}$$

$$= \lambda b + (1-\lambda)b$$

$$\therefore Ax = b$$

and since $x^{(1)} \geq 0, x^{(2)} \geq 0, \lambda \geq 0 \Rightarrow 1-\lambda \geq 0$

$$\therefore x \geq 0$$

$\therefore$ The solution x (is a convex linear combination of $x^{(1)}$ and $x^{(2)}$) is also a feasible solution. Therefore set of all feasible solutions constitute a convex set.

UNIT-II CONCEPT OF ARTIFICIAL VARIABLE

Artifical variable:- If all the constraints are '$\leq$' type, then LPP provides an initial basic feasible solution; but if any one of the constraint of a LPP is $\geq$ or $=$, then a non-negative variable is to be added to the left hand side of the constraint to form an initial basic feasible solution is called an artifical variable.

Artifical Variable Techniques:- we have two methods to solve a LPP which involves artifical variables, they are

1. Big-M method / Penalty Method / charnes Penalty Method.
2. Two-Phase method.

Advantages of Two Phase Method over Big-M Method:-

The advantages of this method over Big-M method are

1. It is easier to calculate as it does not involve 'M' and all are numericals.
2. It can give the solution at the first phase itself if the LPP is infeasible. we need not go through second phase.
3. In the case of a digital computer, it is not possible to get the solution by Big-M where as 2 phase method can be applied.

Big-M Method or Method of Penalities:-

The following steps involved in this method.

Step1. Write the given LPP into its standard form and check whether there exists a starting basic

i) If there is a ready starting basic feasible solution then go to step3

~~ii) If ~~thes~~ there is a~~

ii) If there does not exist a ready starting basic feasible solution then goto Step 2.

Step2. Add artifical variables to the LHS of each that has no obivious starting basic variables.

Step3. Apply simplex method to the modified LPP until the following cases arise

i) Atleast one artifical variable is present in the basis with zero value. In such a case the current optimum basic feasible solution is degenerated.

ii) Atleast one artifical variable is present in the basis with a positive value. In such a case the given LPP doesnot possess any optimum basic feasible solution.

Two-Phase Simplex Method :-

The following steps involved in this method.

Step 1. write the given LPP into its standard form and check whether there exist a starting basic feasible solution.

i) If there is a ready starting basic feasible solution then go to Phase 2.

ii) If there does not exist a ready starting basic feasible solution then go to next step.

Phase - I :-

Step 2. Add artifical variable to the LHS of each equation that lacks the needed starting basic variables. Construct an new objective function by taking all the variables cost is equal to zero and artifical variable cost -1.

$$\text{Max } Z = -A_1 - A_2 \ldots - A_n$$

where A_i $(i = 1, 2 \ldots n)$ are non-negative artifical variables.

i) Max $Z' < 0$ and atleast one artifical variable is present in the ~~basic~~ ~~wi~~ basis with positive value. In such a case the original LPP does not possess any feasible solution.

ii. Max $Z' = 0$ and atleast one artifical variable is present in the basis at ~~to~~ zero value. In such a case the original LPP ~~Pro~~ possess the feasible solution. In order to get basic feasible solution we may proceed to Phase 2.

iii. Max $Z' = 0$ and no artifical variable is present in the basis. In such a case a basic feasible solution to the original LPP has been found, go to Phase 2.

Phase - II :-

Step 3:- Consider the optimum basic feasible solution of Phase I as a starting basic feasible solution for the original LPP. Assign actual co-efficients to the variables in the objective function and a value zero to the artificial variables that appears at zero value in the final simplex table of Phase - I

Apply usual simplex method to the modified simplex table until we get an optimum solution or there is an indication of unbounded solution.

24/5/22

Problems on Big M - Method :-

1Q. Solve the following LPP using big-M method.

$$\text{Min } Z = 2x_1 + 3x_2$$

subject to the constraints : $-x_1 + x_2 \geq 5$

$$x_1 + 2x_2 \geq 6$$

$$\text{and } x_1, x_2 \geq 0$$

Sol. Given Problem is minimization Problem so we convert it into maximization Problem by using $\text{Max } Z^* = -\text{Min } Z$

where $Z^* = -Z$

then we get $\text{Max } Z^* = -\text{Min } Z$

$$= -[2x_1 + 3x_2]$$

$$\text{Max } Z^* = -2x_1 - 3x_2$$

Introduce the surplus variables $x_3, x_4 \geq 0$ and artifical variables $x_5, x_6 \geq 0$ and put the cost of these variables equal to $0, 0, -M, -M$ respectively

The objective function then we get

$$\text{Max } Z^* = -2x_1 - 3x_2 + 0x_3 + 0x_4 - Mx_5 - Mx_6$$

subject of the constraints :- $x_1 + x_2 - x_3 + 0x_4 + x_5 + 0x_6 = 5$

$$x_1 + 2x_2 + 0x_3 - x_4 + 0x_5 + x_6 = 6$$

$$\text{and } x_j \geq 0, j = 1, 2, \ldots 6$$

Matrix form :- $Ax = B$

$$A = \begin{bmatrix} 1 & 1 & -1 & 0 & 1 & 0 \\ 1 & -2 & 0 & -1 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 5 \\ 6 \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_6 \end{bmatrix}$$

$$\sigma = [-2 \quad -3 \quad 0 \quad 0 \quad -M \quad -M]$$

starting table :-

		C_j	-2	-3	0	0	-M	-M
Y_B	C_B	X_B	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6
Y_5	-M	5	1	1	-1	0	1	0
Y_6	-M	6	1	$\boxed{2}$	0	-1	0	1
	$Z^* = -11M$	$Z_j - C_j$	$-2M+2$	$-3M+3$	M	M	0	0

Ratio Min $\{X_B/Y_2\}$

$5/1 = 5$

$6/2 = 3 \leftarrow$

$R_1 \to R_1 - \frac{R_2}{2}$

$R_2 \to \frac{R_2}{2}$

$R_1 - \frac{R_2}{2} = \frac{5}{3} \quad \frac{1}{1/2} \quad \frac{1}{1} \quad \frac{-1}{0} \quad \frac{0}{-1/2} \quad \frac{1}{0}$

$\frac{3}{3} \quad 1/2 \quad 0 \quad -1 \quad 1/2 \quad 1$

		C_j	-2	-3	0	0	-M
Y_B	C_B	X_B	Y_1	Y_2	Y_3	Y_4	Y_5
Y_5	-M	2	$\boxed{1/2}$	0	-1	1/2	1
Y_2	-3	3	1/2	1	0	-1/2	0
	$Z^* = -2M-9$	$Z_j - C_j$	$-\frac{M}{2}+\frac{1}{2}$	0	M	$-\frac{M}{2}+\frac{3}{2}$	0

Ratio Min $\{X_B/Y_1\}$

$2/1/2 \Rightarrow 4 \leftarrow$

$3/1/2 \Rightarrow 6$

$R_2 \to 2R_1$

$R_2 \to R_2 - \frac{1}{2} \times R_1$

		C_j	-2	-3	0	0
Y_B	C_B	X_B	Y_1	Y_2	Y_3	Y_4
Y_1	-2	4	1	0	-2	1
Y_6	-3	1	0	1	1	-1
	$Z^* = -11$	$Z_j - C_j$	0	0	1	1

Since All $Z_j - C_j \geq 0$

Hence we have been reached the optimum Solution and the optimum value of

$Z = -Z^* = -(-11)$

$Z = 11$ for the ~~valu~~ variables $x_1 = 4$ & $x_2 = 1$

23/8/22

2Q. Solve the following LPP by using Big-M method.

$$\text{Max } Z = x_1 + x_2$$

Subject to constraints :- $x_1 - x_2 \geq 0$

$$3x_1 + x_2 \geq 3$$

and $x_1, x_2 \geq 0$

Sol. The given LPP is of maximization type is

$$\text{Max } Z = x_1 + x_2$$

$$x_1 - x_2 \geq 0$$

$$3x_1 + x_2 \geq 3$$

Introduce the surplus variables $x_3, x_4 \geq 0$ and artifical variables $x_5, x_6 \geq 0$ and put the cost of these variables $= 0, 0, -M, -M$ respectively.

The objective function, then we get

$$\text{Max } Z = x_1 + x_2 + 0x_3 + 0x_4 - Mx_5 - Mx_6$$

Subject of constraints :- $x_1 - x_2 - x_3 + 0x_4 + x_5 + 0x_6 = 0$

$$3x_1 + x_2 + 0x_3 - x_4 + 0x_5 + x_6 = 3$$

and $x_j \geq 0 \; j = 1, 2 \ldots 6$

Matrix A form :- $Ax = B$

$$A = \begin{bmatrix} 1 & -1 & -1 & 0 & 1 & 0 \\ 3 & 1 & 0 & -1 & 0 & 1 \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_6 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 1 & 0 & 0 & -M & -M \end{bmatrix}$$

Starting table :-

		C_j	1	1	0	0	-M	-M	Ratio
Y_B	C_B	X_B	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Min $\{X_B/Y_1\}$
Y_5	-M	0	$\boxed{1}$	-1	-1	0	1	0	$\theta_1 = 0$
Y_6	-M	3	3	1	0	-1	0	1	$3/3 = 1$
	$Z = -3M$	$Z_j - C_j$	-4M-1	-1	M	M	0	0	

$R_1 \to R_1$

$R_2 \to R_2 - 3R_1$

		C_j	1	1	0	0	-M	Ratio Min $\{x_B/y_2\}$
y_B	C_B	x_B	y_1	y_2	y_3	y_4	y_6	
y_1	1	0	1	-1	-1	0	0	-
y_6	-M	3	0	4	3	-1	1	3/2
	$z=-3M$	z_j-C_j	0	-4M-2	-1-3M	M	0	

$R_1 \to R_1 + \frac{R_2}{4}$

$R_2 \to \frac{R_2}{4}$

		C_j	1	1	0	0
y_B	C_B	x_B	y_1	y_2	y_3	y_4
y_1	1	3/4	1	0	-1/4	-1/4
y_2	1	3/4	0	1	3/4	-1/4
	$z=3$	z_j-C_j	0	0	1/2	-1/2

Since the most negative Z_4-C_4 is $+1/2$ $-1/2$ and all the elements in y_4 are negative.

$\therefore$ The given LPP has an unbounded solution.

26/5/22

Problem on Two-Phase Simplex Method:-

1Q. Solve the following Problem by using two-phase simplex method.

$$\text{Min } Z = 2x_1 + 3x_2$$

Subject of constraints :- $x_1 + x_2 \geq 5$

$$x_1 + 2x_2 \geq 6$$

and $x_1, x_2 \geq 0$

Sol. The given LPP is minimization.

Therefore convert into maximization Problem

$$\text{Max } Z^* = -\text{Min } Z$$

$$= -[2x_1 + 3x_2]$$

$$= -2x_1 - 3x_2$$

Introduce the surplus variables $x_3, x_4 \geq 0$ and artifical variables $x_5, x_6 \geq 0$ and Put the cost of these variables 0, 0, -1, -1 respectively in the objective function then we get

$$\text{Max } Z^* = -2x_1 - 3x_2 + 0x_3 + 0x_4 - x_5 - x_6$$

Soc :- $x_1 + x_2 - x_3 + 0x_4 + x_5 + 0x_6 = 5$

$$x_1 + 2x_2 + 0x_3 - x_4 + 0x_5 + x_6 = 6$$

and $x_j \geq 0$, $j = 1, 2, \ldots 6$

Matrix form :- $AX = B$

$$A = \begin{bmatrix} 1 & 1 & -1 & 0 & 1 & 0 \\ 1 & 2 & 0 & -1 & 0 & 1 \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_6 \end{bmatrix} \quad B = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$$

$$C = \begin{bmatrix} -2 & -3 & 0 & 0 & -1 & -1 \end{bmatrix}$$

Phase I

Put all the original cost = 0 except artifical variables then we get

$$\text{Max } Z' = -x_5 - x_6$$

Starting table:

		C_j	0	0	0	0	-1	-1
Y_B	C_B	X_B	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6
Y_5	-1	5	1	1	-1	0	1	0
Y_6	-1	6	1	$\boxed{2}$	0	-1	0	1
	$z^1 = -11$	$z_j - C_j$	-2	-3	1	1	0	0

Ratio Min $\{X_B/Y_2\}$

$5/1 = 5$

$6/2 = 3$ ←

$R_1 \to R_1 - \frac{R_2}{2}$

$R_2 \to \frac{R_2}{2}$

		C_j	0	0	0	0	-1	-1
Y_B	C_B	X_B	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6
Y_5	-1	2	$\boxed{1/2}$	0	-1	1/2	1	-1/2
Y_6	0	3	1/2	1	0	-1/2	0	1/2
	$z^1 = -2$	$z_j - C_j$	-1/2	0	1	1/2	0	3/2

Ratio Min $\{X_B/Y_1\}$

$2/1/2 = 4$

$3/1/2 = 6$

$R_2 \to 2R_1$

$R_2 \to R_2 - \frac{1}{2} \cancel{2} R_1$

$R_2 \to R_2 - R_1$

$$R_1 - \frac{R_2}{2} = \begin{array}{cccccc} 5 & 1 & 1 & -1 & 0 & 1 & 0 \\ - & - & - & - & - & - & - \\ 3 & 1/2 & 1 & 0 & -1/2 & 0 & 1/2 \\ \hline 2 & 1/2 & 0 & -1 & 1/2 & 1 & -1/2 \end{array}$$

		C_j	0	0	0	0	-1	-1
Y_B	C_B	X_B	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6
Y_4	0	4	1	0	-2	1	2	-1
Y_2	0	1	0	1	1	-1	-1	1
	$z^1 = 0$	$z_j - C_j$	0	0	0	0	1	1

Phase - II

Consider the last table of phase - I and take the original values then we get

		C_j	-2	-3	0	0	-1	-1
Y_B	C_B	x_B	y_1	y_2	y_3	y_4	y_5	y_6
y_1	-2	4	1	0	-2	1	2	-1
y_2	-3	1	0	1	1	-1	-1	1
$Z^* = -11$		$Z_j - C_j$	0	0	1	1	0	0

All $Z_j - C_j \geq 0$

Hence we have been reached the optimum solution in the optimum value of $Z = -Z^* = -(-11)$

$Z = 11$

for the values $x = 4$ & $x = 1$,,

RQ. Solve the following LPP by using Big M-Method.

$$\text{Max } Z = x_1 + x_2$$

Subject of constraints :- $x_1 - x_2 \geq 0$

$$3x_1 + x_2 \geq 3$$

and $x_1, x_2 \geq 0$

Sol. Introduce the surplus variables $x_3, x_4 \geq 0$ and artificial variables $x_5, x_6 \geq 0$ and put the cost of these variables 0, 0, -1, -1 respectively. The objective function, then we get

$$\text{Max } Z = x_1 + x_2 + 0x_3 + 0x_4 - x_5 - x_6$$

SOC :- $x_1 - x_2 - x_3 + 0x_4 + x_5 + 0x_6 = 3$

$$3x_1 + x_2 + 0x_3 - x_4 + 0x_5 + x_6 = 0$$

and $x_j \geq 0$, $j = 1, 2, \ldots 6$

Matrix form :- $AX = B$

$$A = \begin{bmatrix} 1 & -1 & -1 & 0 & 1 & 0 \\ 3 & 1 & 0 & -1 & 0 & 1 \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_6 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 1 & 0 & 0 & -1 & -1 \end{bmatrix}$$

Phase - I

Put all the original cost = 0 except artifical variables then we get

$$\text{Max } z' = -x_5 - x_6$$

starting table:-

		C_j	0	0	0	0	-1	-1	Ratio $\min\{x_B/y_1\}$
Y_B	C_B	X_B	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	
Y_5	-1	0	$\boxed{1}$	-1	-1	0	1	0	$\leftarrow 0/1 = 0$
Y_6	-1	3	3	1	0	-1	0	1	$3/3 = 1$
	$z' = -3$	$z_j - C_j$	-4	-2	1	1	0	0	

$R_1 \to R_1$

$R_2 \to R_2 - 3R_1$

		C_j	0	0	0	0	-1	-1	Ratio min $\{x_B/y_2\}$
Y_B	C_B	X_B	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	
Y_1	0	0	1	-1	-1	0	1	0	-
Y_6	-1	3	0	$\boxed{4}$	3	-1	-3	1	$3/4 \leftarrow$
	$z' = -3$	$z_j - C_j$	+	-4	-3	1	4	0	

$R_1 \to R_1 + \frac{R_2}{4}$

$R_2 \to \frac{R_2}{4}$

		C_j	0	0	0	0	-1	-1
Y_B	C_B	X_B	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6
Y_1	0	3/4	1	0	-1/4	-1/4	1/4	1/4
Y_2	0	3/4	0	1	3/4	-1/4	-3/4	1/4
	$z' = 0$	$z_j - C_j$	0	0	0	0	1	1

Phase - II

		C_j	1	1	0	0	-1	-1
y_8	C_B	x_B	y_1	y_2	y_3	y_4	y_5	y_6
y_1	1	3/4	1	0	-1/4	-1/4	1/4	1/4
y_2	1	3/4	0	1	3/4	-1/4	-3/4	1/4
	$z^* = 3/2$	$z_j - c_j$	0	0	1/2	-1/2	1/2	3/4

Since the most negative $z_4 - C_4$ is $-\frac{1}{2}$ and all the elements in y_4 are negative.

$\therefore$ The given LPP has an unbounded solution.

28/5/2020

Degeneracy in a Linear Programming Problem :

Degeneracy in a LPP may occur at the initial stage or any other subsequen iteration of the simplex method. The Problem of degeneracy may arise due to the following reasons:

1. If atleast one or more basic variables vanish (i.e. zero) in the initial basic feasible solution.
2. If the minimum ratio $\left\{ \frac{x_{B_i}}{y_{ij}}, y_{ij} > 0 \right\}$ is not unique when finding the Procedure of which vector is leaving from the basis at any iteration of the simplex method.

Cycling :- If the minimum ratio is zero for two or more basic variables, then the solution in one iteration may repeat again after few iterations and/or the subsequent iterations may not improve the value of the objective function. This results the optimum solution may not be obtained, i.e same simplex iterations may be repeated without improving the solution. This concept is known as cyclic.

Method to Resolve Degeneracy :- If the Problem of degeneracy arises at any iteration of the simplex method of linear Programming Problem, the following Procedure can be utilised to resolve the degeneracy in LPP.

Step1 :- First of all identify the vectors for which the minimum ratio $\left\{ \frac{x_{B_i}}{y_{ij}}, y_{ij} > 0 \right\}$ is tied i.e same (not unique). let it be for two vectors in the second and third rows.

Step2 :- Rearrange the columns of the simplex table so that basis matrix come first and then remaining all vectors.

Step3 :- Now find the minimum ratio by using

$$\left\{ \frac{\text{elements of the first column (vector) of basis matrix}}{\text{corresponding elements of pivotal column}} \right\}$$

only for the rows for which minimum ratio is not unique (i.e, for second and third rows).

If this minimum is occurred in the second row (say), then we get the pivotal element corresponding to this column. If this minimum ratio is also not unique, then go to next step.

Step 4:- Compute the minimum ratio by using

$$\left\{ \frac{\text{elements of second column of basis matrix}}{\text{corresponding elements of pivotal column}} \right\}$$

only for the rows for which minimum ratio is not unique in steps 3.

If this minimum ratio is unique, then we get pivotal element in the corresponding column. If this minimum ratio is also not unique, then go to next step.

Step 5 :- Compute the minimum ratio by using

$$\left\{ \frac{\text{elements of third column of basis matrix}}{\text{corresponding elements of pivotal column}} \right\}$$

only for the rows for which minimum ratio is not unique in step 4.

If this minimum ratio is unique, then we get pivotal element in the corresponding column. If this minimum ratio is also not unique, then go to next step.

Step 6 :- Repeat this procedure until a minimum ratio is obtained. After resolving the degeneracy (i.e. the minimum ratio is unique), the simplex method is applied to obtain an optimum basic feasible solution.

8. Resolve degeneracy and solve the following LPP.

Maximize $Z = 5x_1 + 3x_2$

subject to the constraints :-

$$x_1 + x_2 \le 2$$
$$5x_1 + 2x_2 \le 10$$
$$3x_1 + 8x_2 \le 12$$
$$\text{and } x_1, x_2 \ge 0$$

∴ Introduce the slack variable $x_3, x_4, x_5 \ge 0$ and put the cost of these variables is equal to '0' in the objective then we get

$$\text{Max } Z = 5x_1 + 3x_2 + 0x_3 + 0x_4 + 0x_5$$

SOC :-
$$x_1 + x_2 + x_3 + 0x_4 + 0x_5 = 2$$
$$5x_1 + 2x_2 + 0x_3 + x_4 + 0x_5 = 10$$
$$3x_1 + 8x_2 + 0x_3 + 0x_4 + x_5 = 12$$
$$\text{and } x_j \ge 0, j = 1, 2 \ldots 5$$

Matrix form :- $AX = B$

$$A = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 5 & 2 & 0 & 1 & 0 \\ 3 & 8 & 0 & 0 & 1 \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_5 \end{bmatrix} \quad B = \begin{bmatrix} 2 \\ 10 \\ 12 \end{bmatrix} \quad C = \begin{bmatrix} 5 & 3 & 0 & 0 & 0 \end{bmatrix}$$

Starting table :-

		C_j	5	3	0	0	0	Ratio min $\{X_B/Y_1\}$
Y_B	C_B	X_B	Y_1	Y_2	Y_3	Y_4	Y_5	
Y_3	0	2	1	1	1	0	0	$2/1 = 2$
Y_4	0	10	5	2	0	1	0	$10/5 = 2$
Y_5	0	12	3	8	0	0	1	$\frac{12}{3} = 4$
	$Z=0$	$z_j - c_j$	-5	-3	0	0	0	

$Z_1 - C_1$ is the most negative ∴ Y_1 enters the basis.

But min $\{2, 2, 4\} = 2$ but it occurs Y_3 Y_4 ie, min Ratio is not unique is a problem of degeneracy. To resolve the degeneracy rearrange the

simplex table having the basis matrix first, then we get

		C_j	0	0	0	5	3
Y_B	C_B	X_B	Y_3	Y_4	Y_5	Y_1	Y_2
Y_3	0	2	1	0	0	1	1
Y_4	0	10	0	1	0	5	2
Y_5	0	12	0	0	1	3	8
	$Z=0$	z_j-c_j	0	0	0	-5	-3

MRn Ratio $\{Y_3/Y_1\}$

$\Rightarrow \frac{1}{1} = 1$

$\Rightarrow \frac{0}{5} = 0$

$R_2)$ $\frac{2}{2}$ $\frac{1}{0}$ $\frac{0}{1/5}$ $\frac{0}{0}$ $\frac{1}{1}$ $\frac{1}{2/5}$

$0 \quad 1 \quad -1/5 \quad 0 \quad 0 \quad 3/5$

$R_3)$ $\frac{12}{6}$ $\frac{0}{0}$ $\frac{0}{3/5}$ $\frac{1}{0}$ $\frac{3}{3}$ $\frac{8}{6/5}$

$6 \quad 0 \quad -3/5 \quad 1 \quad 0 \quad 34/5$

$R_1 \rightarrow R_1 - R_2/5$

$R_2 \rightarrow R_2/5$

$R_3 \rightarrow R_3 - 3R_2/5$

		C_j	0	0	0	5	3
Y_B	C_B	X_B	Y_3	Y_4	Y_5	Y_1	Y_2
Y_3	0	0	1	-1/5	0	0	3/5
Y_1	5	2	0	1/5	0	1	2/5
Y_5	0	6	0	-3/5	1	0	34/5
	$Z=10$	z_j-c_j	0	1	0	0	-1

MRn Ratio $\{X_B/Y_2\}$

$\Rightarrow \frac{0}{3/5} = 0$

$\frac{2}{2/5} = 5$

$\frac{6}{34/5} = 30/34$

$R_2)$ $\frac{2}{0}$ $\frac{0}{2/3}$ $\frac{1/5}{-2/15}$ $\frac{0}{0}$ $\frac{1}{0}$ $\frac{2/5}{2/5}$

$2 \quad -2/3 \quad 1/3 \quad 0 \quad 1 \quad 0$

$R_3)$ $\frac{6}{0}$ $\frac{0}{34/5}$ $\frac{-3/5}{-34/5}$ $\frac{1}{0}$ $\frac{0}{0}$ $\frac{34/5}{34/5}$

$6 \quad -34/5 \quad 5/3 \quad 1 \quad 0 \quad 0$

$R_1 \rightarrow 5/3 R_1$

$R_2 \rightarrow R_2 - (\frac{2}{5})(5/3)R_1 \Rightarrow R_2 \rightarrow R_2 - 2/3 R_1$

$R_3 \rightarrow R_3 - (34/5)(5/3)R_1 \Rightarrow R_3 \rightarrow R_3 - 34/3 R_1$

		C_j	0	0	0	5	3
Y_B	C_B	X_B	Y_3	Y_4	Y_5	Y_1	Y_2
Y_2	3	0	5/3	-1/3	0	0	1
Y_1	5	2	-2/3	1/3	0	1	0
Y_5	0	6	-34/5	5/3	1	0	0
-	$Z=10$	z_j-c_j	5/3	2/3	0	0	0

All $z_j-c_j \geq 0$

Hence we have been reached the optimum solution and the optimum value of $Z=10$ for the variables $x_1=2$, $x_2=0$.

Duality in LPP :- Every linear programming problem is associated with another linear programming problem. The original problem is called the primal LPP and the other problem is called its dual LPP. The relationship b/w the primal and dual is very useful. The optimum solution of one form (primal or dual) of LPP reveals the optimum solution of the other. Sometimes the dual LPP is easier to solve than the primal LPP. In this case one can solve dual LPP so that the primal LPP.

Primal - Dual Relationship :-

If Primal	Then Dual
1. objective : Maximize	1. objective : Minimize
2. variable x	2. covariable y
3. constraint, i	3. constraint, j
4. constraint i: "=" type	4. constraint j: "=" type
5. variable x_j : unrestricted	5. variable y_i : unrestricted
6. $\leq$ type constraints	6. $\geq$ type constraints
7. ≥ 0 variables	7. $\geq$ inequality constraints
8. $\leq$ inequality constraints	8. ≥ 0 variables

Features / characteristics of Duality Principle :- The characteristics of dual are,

1. Dual of the dual is primal.
2. If either the primal or dual problem has a solution than the other also has a solution and their optimum values are equal.
3. If any of the two problems has an infeasible solution, then the value of the objective function of the other is unbounded.
4. The value of the objective function for any feasible solution of the primal is less than the value of the objective function for any feasible solution of the dual.
5. If either the primal or dual has an unbounded solution, then the solution to the other problem is infeasible.
6. If the primal has a feasible solution, but the dual does not have, then the primal will not have a finite optimum solution and vice versa.

25/3/22 PAPER NAME :- OPERATIONS RESEARCH (OR) (NO Practicals) 800 Marks

Unit-I :- LINEAR PROGRAMMING PROBLEM (LPP)

Operations Research : Meaning and scope of OR. Convex sets and their properties. Definition of general LPP. Formulation of LPP. Solution of LPP by graphical method. Fundamental theorem of LPP. Simplex algorithm.

Unit-II :-

Concept of artificial variables. Big-M/Penalty method and two-phase simplex methods. Concept of degeneracy and resolving it. Concept of duality, duality as LPP. Dual Primal relationship.

Unit-III :- TRANSPORTATION PROBLEM

Definition of transportation problem, TPP as a special case of LPP, Initial basis feasible solutions by North-West Corner Rule, Matrix minimum method and VAM. Optimal solution through MODI tableau and stepping stone method for balanced and unbalanced transportation Problem.

Degeneracy in TP and resolving it. Concept of Transshipment Problem.

Unit-IV :- ASSIGNMENT PROBLEM AND SEQUENCING PROBLEM

Formulation and description of Assignment Problem and its variations. Assignment Problem as special case of TP and LPP. Unbalanced assignment Problem, travelling salesman Problem. Optimal solution using Hungarian method.

Problem of sequencing. Optimal sequence of N Jobs on two and three machines without Passing.

Reference Book :- Operation Research By Kanti Swaroop.

28/3/22

UNIT - III TRANSPORTATION PROBLEM

(2 SAQ's [illegible] from this unit)

- Def. of Transportation Problem.
- Transportation Table
- Mathematical formulation of TP.
- Basic def. of TP
  - Feasible solution
  - Basic Feasible solution
  - Degenerate Basic feasible solution
  - Optimum Basic feasible solution
  - Loop in a transportation table
  - Balanced tP
  - unbalanced tP

- Methods of Find an IBFS
  - North-West Corner rule (NW Rule)
  - Matrix - ~~Mi~~ Minimum Method (MMM)
  - Vogel's Approximation Method (VAM)

- Methods of Finding an Optimum Solution
  - Stepping Stone Method.
  - MODI (Modification) or UV-Method

- Degeneracy in tP.

Def. of TP:- The objective of the transportation problem is to transport various quantities of single homogeneous commodity manufactured and stored at various places called origins to different places called destinations so that the total transportation cost is minimum.

Transportation table (or) Matrix:- let us suppose there are m origins and 'n' destinations. The transportation problem of transporting various quantities of commodities from several origins to different destinations can be explained in the following tabular form called transportation table.

	D_1	D_2	.....	D_j	.....	D_n	Availability (or) Supply
O_1	x_{11} C_{11}	x_{12} C_{12}	.....	x_{1j} C_{1j}	.....	x_{1n} C_{1n}	a_1
O_2	x_{21} C_{21}	x_{22} C_{22}	.....	x_{2j} C_{2j}	.....	x_{2n} C_{2n}	a_2
⋮	⋮						⋮
O_i	x_{i1} C_{i1}	x_{i2} C_{i2}	.....	x_{ij} C_{ij}	.....	x_{in} C_{in}	a_i
⋮	⋮						⋮
O_m	x_{m1} C_{m1}	x_{m2} C_{m2}	.....	x_{mj} C_{mj}	.....	x_{mn} C_{mn}	a_m
Demand (or) requirements (b_j)	b_1	b_2	.....	b_j	.....	b_n	

Where $O_1, O_2 \ldots O_m$ are origins

$D_1, D_2 \ldots D_n$ are destinations

$a_1, a_2 \ldots a_m$ are availabilities at respective origins.

$b_1, b_2 \ldots b_n$ are requirements at respective destinations

x_{ij} = the quantity transported from i^{th} origin to j^{th} destination.

C_{ij} = Cost of transporting one unit of commodity from i^{th} origin to j^{th} destination.

and $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ to be assumed.

Mathematical formulation of tp (or) tp as a special case of LPP :-

The transportation problem can be formulated mathematically as a LPP as follows :

$$\text{Min } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} C_{ij}$$

Subject to the constraints

$$\sum_{j=1}^{n} x_{ij} = a_i \; ; \; i = 1, 2 \ldots m.$$

$$\sum_{i=1}^{m} x_{ij} = b_j \; ; \; j = 1, 2 \ldots n$$

and $x_{ij} \geq 0 \;\; \forall \; i = 1, 2 \ldots m$

$j = 1, 2 \ldots n$

Balanced transportation Problem :- If sum of the availabilities at all origins is equal to the sum of the requirements at all destinations, then the Problem is known as balanced transportation Problem.

i.e for a balanced transportation Problem

we have $\boxed{\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j}$

Unbalanced transportation Problem :- If sum of the availabilities at all origins is not equal to the sum of the requirements at all the destinations, then the Problem is known as unbalanced transportation Problem.

For unbalanced transportation Problem, we have $\boxed{\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j}$

Basic Solution :- A solution obtained by setting 'n' variables equal to zero and solving the remaining 'm' variable is called a basic sol^n. These 'm' variables are called basic variables.

Basic Definitions :-

Feasible Solution :- A set of non-negative values $x_{ij} \geq 0$ $i = 1, 2, \ldots, m$, $j = 1, \ldots, n$ which satisfies all the rim requirements is called feasible solution of the transportation Problem.

Basic feasible solution :- A feasible solution of $m \times n$ transportation Problem is said to be basic feasible solution of the total number of allocations is equal to $m+n-1$

Degenerate Basic feasible Solution :- A feasible solution of $m \times n$ transportation Problem is said to be degenerate basic feasible solution of the total number of allocations are lessthan $m+n-1$

Optimum Basic Feasible Solution :- A basic feasible solution is said to be an optimum basic feasible solution if it minimizes the total transportation cost.

Loop in a transportation table :- An ordered set of four or more cells is said to form a loop in a transportation table if it satisfies the following conditions.

1.) A loop starts and ends in the same cell.
2.) In any single row or in single column only two corner points.
3.) All the corner points should be occupied or basic cells except the first one.

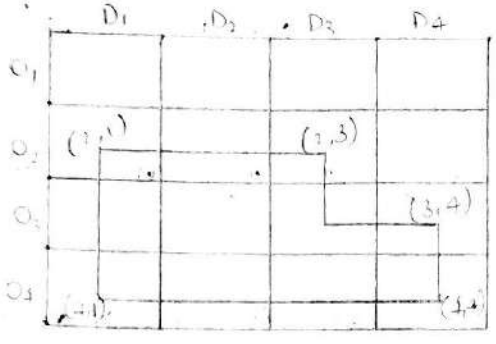
Properties of a Loop :-

1.) Every loop has an even number of cells.

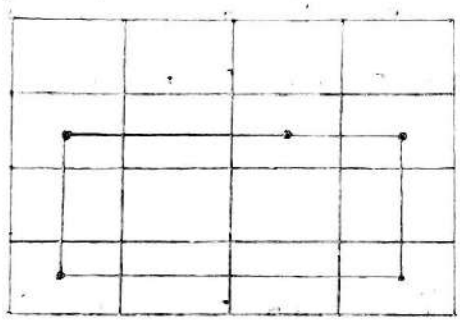
2.) Any loop should have atleast ~~four~~ four cells.

3.) A loop should start and end at the entering unoccupied cell connecting all occupied cells only.

Ex :- i) An order set $L = \{(2,1),(2,3),(3,3),(3,4),(4,4),(4,1)\}$ form a loop. observe the following table.

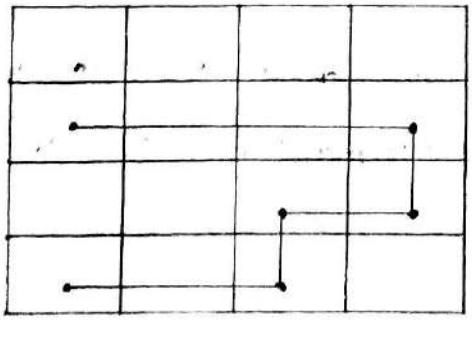


ii)



It does not form a loop

iii)



It does not form a loop

North-West Corner Rule (NW Rule):-

Step 1:- Check whether $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$. If not i.e., $\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j$, then introduce a dummy origin or a dummy destination with the transportation cost, so that they are equal.

Step 2:- Select the upper left i.e., North-West Corner cell of the transportation table and allocate as much as possible quantity. North-West Corner cell is the first cell i.e., (1,1) and allocate $x_{11} = \min\{a_1, b_1\}$

Step 3:- In the allocation of the quantity, we may arise the following three cases.

Case i:- If $a_1 < b_1$, availability at 1st origin is exhausted, then move down to the second row and make next allocation in the cell (2,1)

Case ii:- If $b_1 < a_1$ i.e., requirement at 1st destination is satisfied, then move right horizontally to the second column and make next allocation in the cell (1,2).

Case iii:- If $a_1 = b_1$ i.e., both availability and requirements are satisfied. Allocate $x_{11} = a_1 = b_1$, then move diagonally to the cell (2,2) in second row and second column.

Step 4:- Continue this procedure by moving down towards to the lower right corner until all the rim requirements are satisfied.

4/4/22. Problems On IBFS Methods

North West Corner Rule

1Q. Find IBFS to the following transportation Problem by using North West Corner Rule.

21	16	25	13	11
17	18	14	23	13
32	27	18	41	19
6	10	12	15	

Sol. $\Sigma a_i = 11+13+19 = 43$

$\Sigma b_j = 6+10+12+15 = 43$

$\therefore \Sigma a_i = \Sigma b_j = 43$, There exists an IBFS and it can be solved by using North West Corner Rule.

6 / 21	5 / 16	25	13	~~11~~ 0
17	5 / 18	8 / 14	23	~~13~~ ~~8~~ 0
32	27	4 / 18	15 / 41	~~19~~ ~~15~~ 0
~~6~~ 0	~~10~~ ~~8~~ 0	~~12~~ ~~4~~ 0	~~15~~ 0	

$\therefore$ The Min Transportation cost to the given transportation Problem by using North West Corner Rule.

$Min\ Z = 21\times6 + 16\times5 + 18\times5 + 8\times14 + 18\times4 + 15\times41$

$Min\ Z = 126 + 80 + 90 + 112 + 72 + 615$

$Min\ Z = 1095$

2Q. Find IBFs to the following transportation Problem by using NWCR.

	E	F	G	H	
A	6	8	8	8	30
B	5	9	7	9	40
C	8	7	13	6	50
	35	28	32	25	

Sol. $\Sigma a_i^\circ = 30+40+50 = 120$

$\Sigma b_j^\circ = 35+28+32+25 = 120$

$\therefore \Sigma a_i^\circ = \Sigma b_j^\circ = 120$

	E	F	G	H	
A	30 / 6	8	8	8	~~30~~ 0
B	5 / 5	28 / 9	7 / 7	9	~~40~~ 35 ~~7~~ 0
C	8	7	25 / 13	25 / 6	~~50~~ ~~25~~ 0
	~~35~~ ~~5~~ 0	~~28~~ 0	~~32~~ ~~25~~ 0	~~25~~ 0	

$\therefore$ The Min. Transportation Cost to the given transportation problem by using North West Corner Rule

$\text{Min } Z = 30\times6 + 5\times5 + 28\times9 + 7\times7 + 25\times13 + 25\times6$

$\text{Min } Z = 180 + 25 + 252 + 49 + 325 + 150$

$\text{Min } Z = 981$

CR.

4/7/22

3Q. Find IBFS to the following transportation problem by using North West Corner Rule.

Destinations

	D	E	F	Availability
A	5	1	7	10
B	6	4	6	80
C	3	2	5	15
Reqn.	75	20	50	

Sol. $\Sigma a_i = 10+80+15 = 105$

$\Sigma b_j = 75+20+50 = 145$

$\therefore \Sigma a_i \neq \Sigma b_j$, It is an unbalanced transportation problem, make it balance by introducing ~~dummy~~ ~~origin~~ dummy origin 'x' with an availability 40.

Therefore the modified transportation problem is given by

	D	E	F	
A	[10] 5	1	7	~~10~~ 0
B	[65] 6	[15] 4	6	~~80~~ ~~15~~ 0
C	3	[5] 2	[10] 5	~~15~~ ~~10~~ 0
X	0	0	[40] 0	~~40~~ 0
	~~75~~ ~~65~~ 0	~~20~~ ~~5~~ 0	~~50~~ ~~40~~ 0	

$\therefore$ The Min transportation cost to the given transportation problem by using North West Corner Rule

$Min\,Z = 10\times5 + 65\times6 + 15\times4 + 5\times2 + 10\times5 + 40\times0$

$Min\,Z = 50 + 390 + 60 + 10 + 50 + 0$

$Min\,Z = \cancel{380}\ 560$

4Q. Find IBFS to the following transportation problem by using North West Corner Rule.

	D_1	D_2	D_3	Avo. (a_i)
O_1	8	7	3	60
O_2	3	8	9	70
O_3	11	3	5	80
Rev. (b_j)	50	70	60	

Sol. $\Sigma a_i = 60+70+80 = 210$

$\Sigma b_j = 50+70+60 = 180$

$\because \Sigma a_i \neq \Sigma b_j$, It is an unbalanced transportation problem, make it balance by introducing dummy Destination D_4 with a requirement 30. Therefore the modified transportation problem is given by

	D_1	D_2	D_3	D_4	
O_1	50 / 8	10 / 7	/ 3	/ 0	~~60~~ 10 0
O_2	/ 3	60 / 8	10 / 9	/ 0	~~70~~ 10 0
O_3	/ 11	/ 3	50 / 5	30 / 0	~~80~~ ~~30~~ 0
	~~50~~ 0	~~70~~ ~~60~~ 0	~~60~~ ~~50~~ 0	~~30~~ 0	

$\therefore$ The Min. transportation cost to the given transportation problem by using North West Corner Rule

$\text{Min } Z = 50\times8+10\times7+60\times8+10\times9+50\times5+30\times0$

$= 400+70+480+90+250+0$

$\text{Min } Z = 1290$

Q 5. Find IBFs to the following transportation problem by using North West Corner Rule.

	D_1	D_2	D_3	D_4	
O_1	4	6	8	13	500
O_2	13	11	10	8	700
O_3	14	4	10	13	300
O_4	9	11	13	3	500
	400	350	1050	500	

Sol. $\Sigma a_i = 500+700+300+500 = 2000$

$\Sigma b_j = 400+350+1050+500 = 2300$

$\therefore \Sigma a_i \neq \Sigma b_j$, It is an unbalanced transportation problem, make it balance by introducing dummy origin 'O_5' with a requirement 300. Therefore the modified transportation Problem is given by

	D_1	D_2	D_3	D_4	
O_1	400 / 4	100 / 6	8	13	~~500~~ ~~100~~ 0
O_2	13	250 / 11	450 / 10	8	~~700~~ ~~450~~ 0
O_3	14	4	300 / 10	13	~~300~~ 0
O_4	9	11	300 / 13	200 / 3	~~500~~ ~~200~~ 0
O_5	0	0	0	300 / 0	~~300~~ 0
	~~400~~ 0	~~350~~ ~~250~~ 0	~~1050~~ ~~600~~ ~~300~~ 0	~~500~~ ~~300~~ 0	

$\therefore$ The Min. transportation Cost to the given transportation problem by using North West Corner Rule

$$\text{Min } Z = 400\times4 + 100\times6 + 250\times11 + 450\times10 + 300\times10 + 300\times13 + 200\times3 + 300\times0$$

$$= 1600+600+2750+4500+3000+3900+600+0$$

$$\text{Min } Z = 16950$$

7/04/22

Matrix Minimum Method (OR) Least Cost Entry Method :-

Step1 :- check whether $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$. If not, introduce a dummy origin or a dummy destination with the transportation cost 0, so that they are equal.

Step2 :- Determine the smallest cost in the tp table. Let it be C_{ij} in the cell (i,j). Allocate as much as possible quantity $x_{ij} = \min\{a_i, b_j\}$ in the cell (i,j)

Step 3 :- In the allocation of quantity, we may arise the following three cases

Case i :- If $a_i < b_j$ (i.e., $x_{ij} = a_i$) i.e the availability at i^{th} origin o_i is exhausted then delete i^{th} row

Case ii :- If $a_i > b_j$ (i.e., $x_{ij} = b_j$) i.e., the requirement at j^{th} destination is satisfied, then delete j^{th} column.

Case iii :- If $x_{ij} = a_i = b_j$ i.e., the availability at i^{th} origin as well as requirement at j^{th} destination are satisfied, then cancel either i^{th} row or j^{th} column, but not both.

Step 4 :- Repeat the procedure until all rim requirements are satisfied.

19. Find IBFS to the following transportation problem by using Matrix Minimum Method.

	1	2	3	4	
1	21	16	25	13	11
2	17	18	14	23	13
3	32	27	18	41	19
	6	10	12	15	

Sol. $\Sigma a_i = 11+13+19 = 43$

$\Sigma b_j = 6+10+12+15 = 43$

$\therefore \Sigma a_i = \Sigma b_j$, There exists an IBFS and it can be solved by using Matrix Minimum Method.

The Min. cost value in the cost matrix 13 therefore first allocation is made in the cell (1, 4) as shown in the following table

~~21~~	~~16~~	~~25~~	11 ~~13~~	~~11~~ 0
17	18	14	23	13
32	27	18	41	19
6	10	12	~~15~~ 4	

and the reduced transportation problem table is given by

17	18	12 ~~14~~	23	~~13~~ 1
32	27	~~18~~	41	19
6	10	~~12~~ 0	4	

and the reduced transportation problem is given by

1 ~~17~~	~~18~~	~~23~~	~~10~~
32	27	41	19
~~8~~ 5	10	4	

5 32	10 27	4 41	19
5	10	4	

∴ The Min. transportation cost to the given transportation problem by using Matrix Minimum Method.

∴ $\text{Min}\,Z = 11\times13 + 12\times14 + 17\times1 + 32\times5 + 27\times10 + 41\times4$

$\text{Min}\,Z = 922$

2Q. Find IBFS to the following transportation problem by using Matrix Minimum Method.

	E	F	G	H	
A	6	8	8	8	30
B	5	9	7	9	40
C	8	7	13	6	50
	35	28	32	25	

Sol. $\Sigma a_i = 30 + 40 + 50 = 120$

$\Sigma b_j = 35 + 28 + 32 + 25 = 120$

∴ $\Sigma a_i = \Sigma b_j$, There exists an IBFS and it can be solved by using Matrix Minimum Method.

The Min. cost value in the cost matrix 5 therefore first allocation is made in the cell (B, E) as shown in the following table.

~~6~~	8	8	8	30
35 ~~5~~	9	7	9	~~40~~ 5
~~8~~	7	13	6	50
~~35~~ 0	28	32	25	

and the reduced transportation Problem table is given by

8	8	~~8~~	30
9	7	~~9~~	5
7	13	25 ~~6~~	~~50~~ 25
28	32	~~25~~ 0	

and the reduced transportation Problem table is given by

8	8	30
9	7	5
25 ~~7~~	~~13~~	~~25~~ 0
~~28~~ 3	32	

and the reduced transportation Problem table is given by

8	8	30
~~9~~	5 ~~7~~	~~5~~ 0
3	~~32~~ 27	

3 ~~8~~	27 ~~8~~	~~30~~
~~3~~	~~27~~	

and the reduced transportation Problem table is given by

3 8	27 8	30
3	27	

∴ The Min. transportation Cost to the given transportation Problem by using Matrix Minimum Method.

$$\text{Min } Z = 35\times5+25\times6+25\times7+5\times7+3\times8+27\times8$$

$$\text{Min } Z = 175+150+175+35+24+216 = 775$$

3Q. Find IBFS to the following transportation Problem by using Matrix Minimum Method.

	D	E	F	
A	5	1	7	10
B	6	4	6	80
C	3	2	5	15
	75	20	50	

Sol. $\Sigma a_i = 10 + 80 + 15 = 105$

$\Sigma b_j = 75 + 20 + 50 = 145$

$\therefore \Sigma a_i \neq \Sigma b_j$, It is an unbalanced transportation problem, make it balance by introducing dummy origin 'x' with an availability 40.

The Min. cost value in the cost matrix '0' therefore first allocation is made in the cell (x, D) as shown in the following table.

	D	E	F	
A	5	1	7	10
B	6	4	6	80
C	3	2	5	15
x	40 ~~0~~	~~0~~	~~0~~	~~40~~ 0
	~~75~~ 35	20	50	

and the reduced transportation Problem table is given by

~~5~~	10 ~~1~~	~~7~~	~~10~~ 0
6	4	6	80
3	2	5	15
35	~~20~~ 10	50	

and the reduced transportation problem table is given by

6	~~4~~	6	~~80~~
3	10) ~~2~~	5	~~15~~ 5
35	~~10~~ 0	50	

and the reduced transportation problem table is given by

6	6	80
5) ~~3~~	~~5~~	~~5~~ 0
~~35~~ 30	50	

and the reduced transportation Problem table is given by

30) 6	50) 6	80
30	50	

∴ The Min. Transportation cost to the given transportation problem by using Matrix Minimum Method.

$$\text{Min } Z = 40\times0 + 10\times1 + 10\times2 + 5\times3 + 30\times6 + 50\times6$$

$$\text{Min } Z = \cancel{405}\ 525$$

4Q. Find IBFS to the following transportation problem, by using Matrix Minimum Method.

	D_1	D_2	D_3	
O_1	8	7	3	60
O_2	3	8	9	70
O_3	11	3	5	80
	50	70	60	

Sol. $\Sigma a_i = 60+70+80 = 210$

$\Sigma b_j = 50+70+60 = 180$

$\therefore \Sigma a_i \neq \Sigma b_j$, It is an unbalanced transportation problem, make it balance by introducing dummy destination D_4 with a requirement 30. ~~Therefore~~

The Min. cost value in the cost matrix '0'. Therefore first allocation is made in the cell (O_1, D_4) as shown in the following table.

	D_1	D_2	D_3	D_4	
O_1	8	7	3	30 ~~0~~	~~60~~ 30
O_2	3	8	9	~~0~~	70
O_3	11	3	5	~~0~~	80
	50	70	60	~~30~~ 0	

and the reduced transportation problem table is given by

8	~~7~~	3	30
3	~~8~~	9	70
11	70 ~~3~~	5	~~80~~ 10
50	~~70~~ 0	60	

and the reduced transportation problem table is given by

~~8~~	3	30
50 ~~3~~	9	~~70~~ 20
~~11~~	5	10
~~50~~ 0	60	

30	3	30
20	9	20
10	5	10
	60	

∴ The Min. transportation cost to the given transportation problem by using Matrix Minimum Method

$$\text{Min } Z = 30\times0 + 70\times3 + 50\times3 + 3\times30 + 9\times20 + 5\times10$$

$$\text{Min } Z = 680$$

5Q. Find IBFS to the following transportation problem by using Matrix Minimum Method.

	D_1	D_2	D_3	D_4	
O_1	4	6	8	13	500
O_2	13	11	10	8	700
O_3	14	4	10	13	300
O_4	9	11	13	3	500
	400	350	1050	500	

Sol. $\Sigma a_i = 500 + 700 + 300 + 500 = 2000$

$\Sigma b_j = 400 + 350 + 1050 + 500 = 2300$

∴ $\Sigma a_i \neq \Sigma b_j$, It is an unbalanced transportation problem, make it balance by introducing dummy origin 'o' with a requirement 300.

The Min. cost value in the cost matrix 'o', therefore first allocation is made in the cell (O_5, D_1) as shown in the following table.

	D_1	D_2	D_3	D_4	
O_1	4	6	8	13	500
O_2	13	11	10	8	700
O_3	14	4	10	13	300
O_4	9	11	13	3	500
O_5	800 ~~0~~	~~0~~	~~0~~	~~0~~	~~3000~~
	~~400~~ 100	350	1050	500	

and the reduced transportation problem table is given by

4	6	8	~~13~~ 13	500
13	11	10	~~8~~ 8	700
14	4	10	~~13~~ 13	300
~~9~~	~~11~~	~~13~~	500 ~~3~~	~~500~~ 0
100	350	1050	~~500~~ 0	

and the reduced transportation problem table is given by

4	6	8	13	500
13	11	10	8	700
~~14~~	300 ~~4~~	~~10~~	~~13~~	~~300~~ 0
100	~~350~~ 50	1050	0	

and the reduced transportation problem table is given by

100 ~~4~~	6	8	13	~~500~~ 400
~~13~~	11	10	8	700
~~100~~ 0	50	1050	0	

and the reduced transportation problem table is given by

50 ~~6~~	8	13	~~400~~ 350
~~11~~	10	8	700
~~50~~ 0	1050	0	

8	~~13~~	350
10	0 ~~8~~	700
1050	0	

(350) 8	350
(700) 10	700
1050	

∴ The Min. Transportation cost to the given transportation Problem by using Matrix Minimum Method.

$$\text{Min } Z = 300\times0 + 3\times500 + 300\times4 + 100\times4 + 50\times6 + 8\times350 + 700\times10$$

$$\text{Min } Z = 13200$$

VOGEL'S APPROXIMATION METHOD (VAM):-

Step1 :- Check Whether $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$. If not introduce a dummy origin or a dummy destination with the transportation cost '0' so that they are equal.

Step2 :- Identify smallest and next smallest costs in each row and in each column of the transportation table. Determine difference between smallest and next smallest costs for each row and for each column called Penalities and write them in the Parenthesis against the respective rows and columns.

Step3 :- Identify the largest Penality among all the rows and columns. If tie occurs choose any one. choose minimum cost in the respective row or column has largest Penality. Allocate as much as Possible quantity. If the availability is exhausted, then cancel the respective row or if the requirement is satisfied, then cancel the respective column. If a tie occurs, then cancel either a row or a column, but not both.

Step 4 :- Re consider, the transportation table with the remaining availabilities and requirements. Again calculate penalties for each row and each column and repeat the procedure until all the rim requirements are satisfied.

1Q Find IBFS to the following transportation problem by using vogel's Approximation Method.

Sol.

origin \ Demand	1	2	3	4	Avai.
1	21	16	25	13	11
2	17	18	14	23	13
3	32	27	18	41	19
Req.	6	10	12	15	

Sol. $\Sigma a_i = 11+13+19 = 43$

$\Sigma b_j = 6+10+12+15 = 43$

$\therefore \Sigma a_i = \Sigma b_j$, There exists an IBFS and it can be solved by using vogel's App. Method.

21	16	25	11 / 13	11 0	(3)	–	–	–	–
6 / 17	3 / 18	14	4 / 23	13 9 3 0	(3)	(3)	(3)	(4)	(18) (18)
32	7 / 27	12 / 18	41	19 7 0	(9)	(9)	(9)	(9)	(27) –
6 0	10 3 0	12 0	15 4 0						
(4)	(2)	(4)	(10)						
(15)	(9)	(4)	(18)						
(15)	(9)	(4)	–						
–	(9)	(4)	–						
–	(9)	–	–						
–	(18)	–	–						

$\therefore$ The Min. transportation cost to the given transportation problem by using VAM method

$$\text{Min } Z = 11\times13+6\times17+3\times18+4\times23+7\times27+12\times18$$

$$\text{Min } Z = 796$$

Q. Find IBFS to the following transportation problem by using VAM Method.

	E	F	G	H	
A	6	8	8	8	30
B	5	9	7	9	40
C	8	7	13	6	50
	35	28	32	25	

Sol. $\Sigma a_i^\circ = 30 + 40 + 50 = 120$

$\Sigma b_j^\circ = 35 + 28 + 32 + 25 = 120$

$\therefore \Sigma a_i^\circ = \Sigma b_j^\circ$, There exists an IBFS ~~to~~ and it can be solved by using VAM.

/ / 6	3 / 8	27 / 8	/ 8	~~30~~ 0	(2)	(0)	(0)	(0)	(0) (8)
35 / 5	/ / 9	5 / 7	/ 9	~~40~~ ~~5~~ 0	(2)	(2)	(2)	(2)	— —
/ 8	25 / 7	/ 13	25 / 6	~~50~~ ~~25~~ 0	(1)	(1)	(6)	—	— —
~~35~~ 0	~~28~~ ~~3~~ 0	~~32~~ ~~27~~ 0	~~25~~ 0						

(1)	(1)	(1)	(2)
—	(1)	(1)	(2) ↑
—	(1)	(1)	—
—	(1)	(1)	—
—	(8)	(8)	—
—	(8)	—	—

$\therefore$ The Min transportation cost to the given transportation Problem by using VAM Method

$\text{Min } Z = 3\times8 + 27\times8 + 35\times5 + 5\times7 + 25\times7 + 25\times6$

$\text{Min } Z = 775$

3Q. Find IBFS to the following transportation problem by using VAM.

	D	E	F	
A	5	1	7	10
B	6	4	6	80
C	3	2	5	15
	75	20	50	

Sol. $\Sigma a_i = 10 + 80 + 15 = 105$

$\Sigma b_j = 75 + 20 + 50 = 145$

$\therefore \Sigma a_i \neq \Sigma b_j$, It is an unbalanced transportation problem, make it balance by introducing dummy origin 'x' with an availability 40.

	D	E	F							
A	/ ~~8~~	10 / 1	/ ~~7~~	~~10~~ 0	(4)	(4)	–	–	–	–
B	60 / ~~6~~	10 / ~~4~~	10 / ~~6~~	~~80~~ ~~20~~ ~~10~~ 0	(2)	(2)	(2)	(2)	(2)	(4)
C	15 / 3	/ / 2	/ / ~~5~~	~~15~~ 0	(1)	(1)	(1)	–	–	–
x	/ / 0	/ ~~0~~	40 / ~~0~~	~~40~~ 0	(0)	←	–	–	–	–
	~~75~~ ~~60~~ 0	~~20~~ ~~10~~ 0	~~50~~ ~~10~~ 0							
	(3)	(1)	(5)↑							
	(2)	(1)	(1)							
	(3)↑	(2)	(1)							
	(6)↑	(4)	(6)							
	–	(4)	(6)↑							
	–	(4)	–							

$\therefore$ The Min. transportation cost to the given transportation problem by using VAM.

$$\text{Min } Z = 10\times1 + 6\times60 + 10\times4 + 10\times6 + 15\times3 + 40\times0$$

$$\text{Min } Z = 515$$

4Q. Find IBFS to the following transportation Problem by using VAM.

	D_1	D_2	D_3	
O_1	8	7	3	60
O_2	3	8	9	70
O_3	11	3	5	80
	50	70	60	

Sol. $\Sigma a_i = 60 + 70 + 80 = 210$

$\Sigma b_j = 50 + 70 + 60 = 180$

$\therefore \Sigma a_i \neq \Sigma b_j$, It is an unbalanced transportation problem, make it balance by introducing dummy destination D_4 with a requirement 30.

	D_1	D_2	D_3	D_4						
O_1	8	7	60 3	0	~~60~~ 0	(3)	(3)	(3)	(3)	(3)
O_2	50 3	8	9	20 0	~~70~~ ~~20~~ 0	(3)	(8)	–	–	–
O_3	11	70 3	5	10 0	~~80~~ ~~10~~ 0	(3)	(3)	(3)	(5)	–
	~~50~~ 0	~~70~~ 0	~~60~~ 0	~~30~~ ~~10~~ 0						

(5)↑	(4)	(2)	(0)
–	(4)	(2)	(0)
–	(4)↑	(2)	(0)
–	–	(2)	(0)
–	–	(3)	(0)

$\therefore$ The Min. transportation cost to the given transportation problem by using VAM

$$\text{Min } Z = 60 \times 3 + 50 \times 3 + 20 \times 0 + 70 \times 3 + 10 \times 0$$

$$\text{Min } Z = 540$$

5Q. Find IBFs to the following transportation Problem by using VAM.

	D_1	D_2	D_3	D_4	
O_1	4	6	8	13	500
O_2	13	11	10	8	700
O_3	14	4	10	13	300
O_4	9	11	13	3	500
	400	350	1050	500	

Sol. $\Sigma a_i = 500 + 700 + 300 + 500 = 2000$

$\Sigma b_j = 400 + 350 + 1050 + 500 = 2300$

$\therefore \Sigma a_i \neq \Sigma b_j$. It is an unbalanced transportation Problem, make it balance by introducing dummy origin 'o' with a requirement 300

	D_1	D_2	D_3	D_4								
O_1	400 / 4	50 / 6	50 / 8	/ 13	~~500~~ ~~100~~ ~~50~~ 0	(2)	(2)	(2)	(2)	(2)	(5)	(5) -
O_2	/ 13	/ 11	700 / 10	/ 8	~~700~~ 0	(2)	(2)	(2)	(2)	(2)	(2)	(2) (2)
O_3	/ 14	300 / 4	/ 10	/ 13	~~300~~ 0	(6)	(6)	(6)	(6)	-	-	- -
O_4	/ 9	/ 11	/ 13	500 / 3	~~500~~ 0	(6)	(6)	-	-	-	-	- -
O_5	/ 0	/ 0	300 / 0	/ 0	~~300~~ 0	(0)	-	-	-	-	-	- -
	~~400~~ 0	~~350~~ ~~50~~ 0	~~1050~~ ~~750~~ ~~700~~ 0	~~500~~ 0								
	(4)	(4)	(8)↑	(3)								
	(5)	(2)	(2)	(5)								
	(9)↑	(2)	(2)	(5)								
	-	(2)	(2)	(5)								
	-	(5)↑	(2)	(5)								
	-	-	(2)	(5)								
	-	-	(10)↑	(8)								

$\therefore$ The Min. transportation cost to the given transportation Problem by using VAM.

$$\text{Min } Z = 400\times4 + 6\times50 + 8\times50 + 700\times10 + 4\times300 + 500\times3 + 300\times0$$

$$\text{Min } Z = 12000$$

MODI Method (or) UV - Method :-

Step1 :- Find IBFS by using any of three methods.

Step2 :- Check the no. of occupied cells. If there are less than $m+n-1$, there exists degeneracy and we introduce a very small positive assignment of $\epsilon(\simeq 0)$ in suitable independent positions, so that the no. of occupied cells is exactly equal to $m+n-1$.

Step3 :- For each occupied cell in the current solution, solve the system of equations $u_i+v_j=c_{ij}$, starting initially with some $u_i=0$ or $v_j=0$ and entering successively the values of u_i and v_j in the transportation table margins.

Step4 :- Compute the net - evaluations.

$z_{ij}-c_{ij} = u_i+v_j-c_{ij}$ for all unoccupied basic cells and enter them in the lower left corner of the corresponding cells.

Step5 :- Examine the sign of each $z_{ij}-c_{ij}$. If all $z_{ij}-c_{ij} \leq 0$ then the current basic feasible solution is an optimum solution. If atleast one $z_{ij}-c_{ij} > 0$, select the unoccupied cell, having the largest positive net evaluation to enter the basis.

Step6 :- let the unoccupied cell (r,s) enter the basis. Allocate an unknown quantity say θ to the cell (r,s). Identify a loop that starts and ends at the cell (r,s) and connects some of the basic cells. Add and subtract θ to and from the transition cells of the loop in such a way that the rim requirements remain satisfied.

Step7 :- Assign a maximum value to θ in such a way that the value of one basic variable becomes zero and the other basic variables remain non-negative. The basic cell where allocation has been reduced to zero, leaves the basis.

Step8 :- Return to Step3 and repeat the process until an optimum basic feasible solution has been obtained.

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1Q. Find the optimum solution to the following transportation table.

	D_1	D_2	D_3	D_4	
O_1	1	2	1	4	30
O_2	3	3	2	1	50
O_3	4	2	5	9	20
	20	40	30	10	

Sol. i) $\Sigma a_i = 30+50+20 = 100$

$\Sigma b_j = 20+40+30+10 = 100$

$\therefore \Sigma a_i = \Sigma b_j$, There exists an IBFS and it can be solved by using Vogel's Approximation Method.

20 / 1	/ 2	10 / 1	/ 4	~~30~~ 10 0	(0)	(0)	(1)	(1)
/ 3	20 / 3	20 / 2	10 / 1	~~50~~ 40	(1)	(1)	(1)	(1)
/ 4	20 / 2	/ 5	/ 9	~~20~~ 0	(2)	(2)	(3)	(–)
~~20~~ 0	~~40~~ 20	~~30~~ 20	~~10~~ 0					
(2)	(0)	(1)	(3)					
(2)	(0)	(1)	(–)					
(–)	(0)	(1)	(–)					
(–)	(1)	(1)	(–)					

$\therefore$ The Min. transportation cost to the given transportation problem by using VAM method.

$Min\ Z = 1\times20 + 1\times10 + 3\times20 + 2\times20 + 1\times10 + 2\times20$

$= 20+10+60+40+10+40$

$Min\ Z = 180$

ii) No. of occupied cells = 6

$m+n-1 = 3+4-1 = 6$

$\therefore$ No. of occupied cells = $m+n-1$.

iii) System of equation for occupied cells:-

$$\boxed{U_i + V_j = C_{ij}}$$

					U_i
	20 / 1	2	10 / 1	4	$U_1 = 0$ (Assume)
	3	20 / 3	20 / 2	10 / 1	$U_2 = 1$
	4	20 / 2	5	9	$U_3 = 0$
V_j	$V_1 = 1$	$V_2 = 2$	$V_3 = 1$	$V_4 = 0$	

$U_1 + V_1 = C_{11} = 0 + V_1 = 1 \Rightarrow V_1 = 1$

$U_1 + V_3 = C_{13} = 0 + V_3 = 1 \Rightarrow V_3 = 1$

$U_2 + V_3 = C_{23} = U_2 + 1 = 2 \Rightarrow U_2 = 1$

$U_2 + V_4 = C_{24} = 1 + V_4 = 1 \Rightarrow V_4 = 0$

$U_2 + V_2 = C_{22} = 1 + V_2 = 3 \Rightarrow V_2 = 2$

$U_3 + V_2 = C_{32} = U_3 + 2 = 2 \Rightarrow U_3 = 0$

iv) Net-evaluations for unoccupied cells:-

$$\boxed{Z_{ij} - C_{ij} = U_i + V_j - C_{ij}}$$

	D_1	D_2	D_3	D_4	
O_1	20 / 1	2 / 0	10 / 1	4 / -4	$U_1=0$
O_2	3 / -1	20 / 3	20 / 2	10 / 1	$U_2=1$
O_3	4 / -3	20 / 2	5 / -4	9 / -9	$U_3=0$
	$V_1=1$	$V_2=2$	$V_3=1$	$V_4=0$	

$(O_1 D_2) = 0 + 2 - 2 = 0$

$(O_1 D_4) = 0 + 0 - 4 = -4$

$(O_2 D_1) = 1 + 1 - 3 = -1$

$(O_3 D_1) = 0 + 1 - 4 = -3$

$(O_3 D_3) = 0 + 1 - 5 = -4$

$(O_3 D_4) = 0 + 0 - 9 = -9$

v) All $Z_{ij} - C_{ij} \leq 0$

Hence we have been reached the optimum solution.

$\therefore$ Min $Z = 180$

Q. Find the optimum solution to the following transportation table.

	1	2	3	4	
1	31	16	25	13	11
2	17	18	14	23	13
3	32	27	18	41	19
	6	10	12	15	

Sol. i) $\Sigma a_i = 11+13+19 = 43$

$\Sigma b_j = 6+10+12+15 = 43$

$\therefore \Sigma a_i = \Sigma b_j$, There exists an IBFS and it can be solved by using VAM method.

31	16	25	11 13	~~11~~ 0	(3)	–	–	–
6 17	3 18	14	4 23	~~13~~ ~~9~~ ~~3~~ 0	(3)	(3)	(3)	(4)
32	7 27	12 18	41	~~19~~ 7	(9)	(9)	(9)	(9)
~~6~~ 0	10	~~12~~ 0	~~15~~ ~~4~~ 0					

(4)	(2)	(4)	(10)
(15)	(9)	(4)	(18)
(15)	(9)	(4)	–
–	(9)	(4)	–

$\therefore$ The Min transportation cost to the given transportation problem by using VAM method.

$$\text{Min } Z = 11\times13 + 6\times17 + 3\times18 + 4\times23 + 7\times27 + 12\times18$$

$$\text{Min } Z = 796$$

ii) No. of occupied cells = 6

$$m+n-1 = 3+4-1 = 6$$

$\therefore$ No. of occupied cells $= m+n-1$

iii) System of equation for occupied cells:-

$$\boxed{U_i + V_j = C_{ij}}$$

21	16	25	11 / 13	$U_1 = 0$ (Assume)
6 / 17	3 / 18	14	4 / 23	$U_2 = 10$
32	7 / 27	12 / 18	41	$U_3 = 19$
$V_1 = 7$	$V_2 = 8$	$V_3 = -1$	$V_4 = 13$	

$U_1 + V_4 = C_{14} = 0 + V_4 = 13 \Rightarrow V_4 = 13$

$U_2 + V_4 = C_{24} = U_2 + 13 = 23 \Rightarrow U_2 = 10$

$U_2 + V_1 = C_{21} = 10 + V_1 = 17 \Rightarrow V_1 = 7$

$U_2 + V_2 = C_{22} = 10 + V_2 = 18 \Rightarrow V_2 = 8$

$U_3 + V_2 = C_{32} = U_3 + 8 = 27 \Rightarrow U_3 = 19$.

$U_3 + V_3 = C_{33} = 19 + V_3 = 18 \Rightarrow V_3 = -1$

iv) Net-evaluations for unoccupied cells:-

$$\boxed{Z_{ij} - C_{ij} = U_i + V_j - C_{ij}}$$

	1	2	3	4	
1	21 / -14	16 / -8	25 / -26	11 / 13	$U_1 = 0$
2	6 / 17	3 / 18	14 / -5	4 / 23	$U_2 = 10$
3	32 / -6	7 / 27	12 / 18	41 / -9	$U_3 = 19$
	$V_1 = 7$	$V_2 = 8$	$V_3 = -1$	$V_4 = 13$	

$(1,1) = 0 + 7 - 21 = -14$

$(1,2) = 0 + 8 - 16 = -8$

$(1,3) = 0 - 1 - 25 = -26$

$(2,3) = 10 - 1 - 14 = -5$.

$(3,1) = 19 + 7 - 32 = -6$

$(3,4) = 10 + 13 - 41 = -9$

v) All $Z_{ij} - C_{ij} \leq 0$

Hence we have been reached the optimum solution

$Min\ Z = 796$

3Q. Find the optimum solution to the following transportation table.

	E	F	G	H	
A	6	8	8	8	30
B	5	9	7	9	40
C	8	7	13	6	50
	35	28	32	25	

Sol. i) $\Sigma a_i = 30+40+50 = 120$

$\Sigma b_j = 35+28+32+25 = 120$

$\therefore \Sigma a_i = \Sigma b_j$, There exists an IBFS and it can be solved by using VAM method

6	3 / 8	27 / 8	8	30	(2) (0) (0) (0)
35 / 5	9	5 / 7	9	~~40~~ 0	(2) (2) (2) (2)
8	25 / 7	13	25 / 6	~~50~~ ~~25~~ 0	(1) (1) (6) –
~~35~~ 0	~~28~~ 3	~~32~~ 27	~~25~~ 0		

(1)	(1)	(1)	(2)
–	(1)	(1)	(2)
–	(1)	(1)	–
–	(1)	(1)	–

$\therefore$ The Min. transportation cost to the given transportation cost by using VAM

$\text{Min } Z = 3\times8 + 27\times8 + 35\times5 + 5\times7 + 25\times7 + 25\times6$

$\text{Min } Z = 775$

ii) No. of occupied cells = 6

$m+n-1 = 3+4-1 = 6$

$\therefore$ No. of occupied cells $= m+n-1$

iii) System of equations for occupied cells:-

$$\boxed{U_i + V_j = C_{ij}}$$

6	3 / 8	27 / 8	8	$U_1 = 0$
35 / 5	9	5 / 7	9	$U_2 = -1$
8	25 / 7	13	25 / 6	$U_3 = -1$
$V_1 = 6$	$V_2 = 8$	$V_3 = 8$	$V_4 = 7$	

$U_1 + V_2 = C_{12} = 0 + V_2 = 8 \Rightarrow V_2 = 8$

$U_1 + V_3 = C_{13} = 0 + V_3 = 8 \Rightarrow V_3 = 8$

$U_2 + V_3 = C_{23} = U_2 + 8 = 7 \Rightarrow U_2 = -1$

$U_2 + V_1 = C_{21} = -1 + V_1 = 5 \Rightarrow V_1 = 6$

$U_3 + V_2 = C_{32} = U_3 + 8 = 7 \Rightarrow U_3 = -1$

$U_3 + V_4 = C_{34} = -1 + V_4 = 6 \Rightarrow V_4 = 7$

iv) Net-evaluations for unoccupied cells:-

$$\boxed{Z_{ij} - C_{ij} = U_i + V_j - C_{ij}}$$

	E	F	G	H	
A	6 / 0	3 / 8	27 / 8	8 / -1	$U_1 = 0$
B	35 / 5	9 / -2	5 / 7	9 / -3	$U_2 = -1$
C	8 / -3	25 / 7	13 / -6	25 / 6	$U_3 = -1$
	$V_1 = 6$	$V_2 = 8$	$V_3 = 8$	$V_4 = 7$	

$(A, E) = 0 + 6 - 6 = 0$

$(A, H) = 0 + 7 - 8 = -1$

$(B, F) = -1 + 8 - 9 = -2$

$(B, H) = -1 + 7 - 9 = -3$

$(C, E) = -1 + 6 - 8 = -3$

$(C, G) = -1 + 8 - 13 = -6$

v) All $Z_{ij} - C_{ij} \leq 0$

Hence we have been reached the optimum solution.

$\text{Min } Z = 775$

4Q. Find the optimum solution to the following transportation problem.

	D_1	D_2	D_3	D_4	
O_1	6	4	1	5	14
O_2	8	9	2	7	16
O_3	4	3	6	9	5
	6	10	15	4	

Sol. i) $\Sigma a_i = 14+16+5 = 35$

$\Sigma b_j = 6+10+15+4 = 35$

$\therefore \Sigma a_i = \Sigma b_j$, There exists an IBFS and it can be solved by using VAM.

0	6	10	4		1	4	5	14 ~~4~~ 0	(3) (1) (1) (1)
1	8		9	15	2		7	~~16~~ 1	(5) (1) (1) (1)
5	4		3		6		9	~~5~~ 0	(1) (1) – –
	~~6~~ 1		~~10~~ 0		~~15~~ 0		~~4~~ 0		
	(2)		(1)		(1)		(2)		
	(2)		(1)		–		(2)		
	(2)		(5)		–		(2)		
	(2)		–		–		(2)		

$\therefore$ The Min. transportation cost to the given transportation cost by using VAM

$Min Z = \cancel{60} \times 6\times0 + 4\times10 + 4\times5 + 1\times8 + 15\times2 + 5\times4$

$Min Z = 118$

ii) No. of occupied cells = 6

$m+n-1 = 3+4-1 = 6$

$\therefore$ No. of occupied cells = $m+n-1$

iii) System of eq. for occupied cells:-

$u_i + v_j = C_{ij}$

0 / 6	10 / 4	1 / -1	4 / 5	$u_1 = 0$
1 / 8	9 / -3	15 / 2	7 / 0	$u_2 = 2$
5 / 4	3 / -1	6 / -8	9 / -6	$u_3 = -2$
$v_1 = 6$	$v_2 = 4$	$v_3 = 0$	$v_4 = 5$	

iv) Net evaluations for unoccupied cells :-

$$z_{ij} - C_{ij} = u_i + v_j - C_{ij}$$

v) All $z_{ij} - C_{ij} \leq 0$

Hence we have been reached the optimum solution.

$\text{Min } Z = 118$

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5Q. Find the optimum solution to the following transportation problem by using MODI.

	D_1	D_2	D_3	
O_1	2	4	1	40
O_2	6	3	2	50
O_3	4	5	6	20
O_4	3	2	1	30
O_5	5	2	5	10
	50	60	40	

Sol. i) $\Sigma a_i = 40 + 50 + 20 + 30 + 10 = 150$

$\Sigma b_j = 50 + 60 + 40 = 150$

$\therefore \Sigma a_i = \Sigma b_j$, There exists an IBFS and it can be solved by using VAM.

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(0) 2	4	(40) 1	~~40~~ 0	(1)	(1)	(2)
(0) 6	(50) 3	~~2~~	~~50~~ 0	(1)	(1)	(3)
(20) 4	~~5~~	~~6~~	20	(1)	(1)	(1)
(30) 3	2	~~1~~	30	(1)	(1)	(1)
~~5~~	(10) ~~2~~	~~5~~	~~10~~ 0	(3)	–	–
50	~~60~~ ~~50~~ 0	~~40~~ 0				
(1)	(0)	(0)				
(1)	(1)	(0)				
(1)	(1)	–				

$$\text{Min } Z = 2\times0 + 40\times1 + 6\times0 + 3\times50 + 4\times20 + 3\times30 + 10\times2$$

$$\text{Min } Z = 380$$

ii) No. of occupied cells = 7

$m+n-1 = 5+3-1 = 7$

$\therefore$ No. of occupied cells $= m+n-1$

iii) System of evaluation for occupied cells :-

$$U_i + V_j = C_{ij}$$

2 / 1	4 / -4	(40) 1	$U_1 = 0$
(0) 6	(50) 3	2 / 2	$U_2 = 3$
(20) 4	5 / -4	6 / -4	$U_3 = 1$
(30) 3	2 / -2	(0) 1	$U_4 = 0$
5 / 0	(10) 2	5 / -2	$U_5 = 2$
$V_1 = 3$	$V_2 = 0$	$V_3 = 1$	

iv) Net-evaluations for unoccupied cells :-

$$Z_{ij} - C_{ij} = U_i + V_j - C_{ij}$$

v.) $Z_{23} - C_{23}$ is the most Positive (2).

$\therefore Z_{23} - C_{23}$ enters into the basis.

vi) Assign an unknown quantity 'θ' to the cell (2,3) and start a loop that starts and also ends in the same cell connecting some of the occupied cells.

2 / 1	4 / -4	40 / 1	$u_1 = 0$
0 / 6-θ	50 / 3	θ / 2	$u_2 = 3$
20 / 4	5 / -4	6 / -4	$u_3 = 1$
30 / θ 3	2 / -2	0 / -θ 1	$u_4 = 0$
5 / 0	10 / 2	5 / -2	$u_5 = 2$
$v_1 = 3$	$v_2 = 0$	$v_3 = 1$	

vii) Find the min value in "$-\theta$'s" and the min. value is '0', then the modified transportation table is given by

θ / 1	4 / -2	40 / -θ 1	$u_1 = 0$
6 / -2	50 / 3	0 / θ	$u_2 = 1$
20 / 4	5 / -2	6 / -4	$u_3 = 1$
30 / θ 3	θ / 0	0 / θ 1	$u_4 = 0$
5 / -2	10 / 2	5 / -4	$u_5 = 0$
$v_1 = 3$	$v_2 = 2$	$v_3 = 1$	

viii) Find the min value in "$-\theta$'s" and the min value is '30', then the modified transportation table is given by

30 / θ	4 / -2	10 / 1	$u_1 = 0$
6 / -3	50 / 3	0 / 2	$u_2 = 1$
20 / 4	5 / -1	6 / -3	$u_3 = 2$
3 / -1	2 / 0	30 / 1	$u_4 = 0$
5 / -3	10 / 2	5 / -4	$u_5 = 0$
$v_1 = 2$	$v_2 = 2$	$v_3 = 1$	

$\text{Min } Z = 2\times30 + 10\times1 + 50\times3 + 0\times2 + 20\times4 + 30\times1 + 10\times2$

$= 350$

6Q. Find the optimum solution to the following transportation Problem by using MODI Method.

	D_1	D_2	D_3	D_4	
O_1	19	30	50	10	7
O_2	70	30	40	60	9
O_3	40	8	70	20	18
	5	8	7	14	

Sol. i) $\Sigma a_i = 7+9+18 = 34$

$\Sigma b_j = 5+8+7+14 = 34$

$\therefore \Sigma a_i = \Sigma b_j$, There exists an IBFS and it can be solved by using VAM.

5 / 19	30	50	2 / 10	~~7~~ ~~2~~ 0	(9)	(9)	(40)	(40)
70	30	7 / 40	2 / 60	9	(10)	(20)	(20)	(20)
40	8 / 8	70	10 / 20	~~18~~ ~~10~~ ~~5~~ 0	(12)	(20)	(50)	–
~~5~~ 0	~~8~~ 0	7	~~14~~ ~~7~~ 2					
(21)	(22)	(10)	(10)					
(21)	–	(10)	(10)					
–	–	(10)	(10)					
–	–	(10)	(50)					

$\therefore$ The Min. transportation cost to the given transportation cost by using VAM

$Min\ Z = 19\times5+2\times10+7\times40+2\times60+8\times8+10\times20$

$= 95+20+280+120+64+200$

$Min\ Z = 779$

ii) No. of occupied cells = 6

$m+n-1 = 3+4-1 = 6$

$\therefore$ No. of occupied cells $= m+n-1$

iii) System of equations for occupied cells:-

$$u_i + v_j = C_{ij}$$

5 / 19	30 / -32	50 / -60	2 / 10	$u_1 = 0$
70 / -1	30 / 18	7 / 40	2 / 60	$u_2 = 50$
40 / -11	8 / 8	70 / -70	10 / 20	$u_3 = 10$
$v_1 = 19$	$v_2 = -2$	$v_3 = -10$	$v_4 = 10$	

iv) Net evaluations for unoccupied cells:-

$$Z_{ij} - C_{ij} = u_i + v_j - C_{ij}$$

v) Since $Z_{22} - C_{22}$ is the only '+ve'

$\therefore$ cell (2,2) enters into the basis

vi) Assign an unknown quantity θ to the cell (2,2) and form a loop that starts and also ends in the cell (2,2) connecting some of the occupied cells.

5 / 19	30 / -32	50 / -60	2 / 10	$u_1 = 0$
70 / -1	30 / 18	7 / 40	2 / 60	$u_2 = 50$
40 / -11	8 / 8	70 / -70	10 / 20	$u_3 = 10$
$v_1 = 19$	$v_2 = -2$	$v_3 = -10$	$v_4 = 10$	

vii) The Min. value of $\theta = \{2, 8\} = 2$, then the modified transportation Problem is given by

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5 / 19	30 / -32	50 / -42	2 / 10	$u_1 = 0$
70 / -19	2 / 30	7 / 40	60 / -18	$u_2 = 32$
40 / -11	6 / 8	70 / -52	12 / 20	$u_3 = 10$
$v_1 = 19$	$v_2 = -2$	$v_3 = 8$	$v_4 = 10$	

viii) All $Z_{ij} - C_{ij} \leq 0$

Hence We have been reached the optimum solution and the optimum value of z is given by

$$\text{Min } Z = 19\times5 + 2\times10 + 2\times30 + 7\times40 + 6\times8 + 12\times20$$

$$= 95 + 20 + 60 + 280 + 48 + 240$$

$$\text{Min } Z = 743$$

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Problems on Degeneracy in Transportation Problem :-

7Q. Find optimum solution to the given transportation Problem by using MODI or UV-Method.

	D_1	D_2	D_3	D_4	
O_1	8	10	7	6	50
O_2	12	9	4	7	40
O_3	9	11	10	8	30
	25	32	40	23	

Sol. i) $\Sigma a_i = 50 + 40 + 30 = 120$

$\Sigma b_j = 25 + 32 + 40 + 23 = 120$

$\therefore \Sigma a_i = \Sigma b_j$, There exists an IBFS Solution and it can be solved by using VAM

25 / 8	2 / 10	7	23 / 6	~~50~~ ~~27~~ 2	(1)	(2)	(2)	(2)
12	9	40 / 4	7	~~40~~ 0	(3)	(2)	(3)	–
9	30 / 11	10	8	30	(1)	(1)	(2)	(2)
~~25~~ 0	32	~~40~~ 0	~~23~~ 0					
(1)	(1)	(3)	(1)					
(1)	(1)	–	(1)					
(1)	(1)	–	–					
(1)	(1)	–	–					

$\therefore$ Min $Z = 8\times25 + 2\times10 + 23\times6 + 40\times4 + 11\times30$

Min $Z = 848$.

ii) No. of occupied cells $= 5$

$m+n-1 = 3+4-1 = 6$

$\therefore$ No. of occupied cells $\neq m+n-1$, there exists a degeneracy introduce a very small positive assignment 'ϵ' which is equal to '0' ($\epsilon (\simeq 0)$) in a suitable independent position (min. value) then the modified transportation table is given by

25 / 8	2 / 10	ϵ / 7	23 / 6
12	9	40 / 4	7
9	30 / 11	10	8

iii) System of equations for occupied cells:-

$$U_i + V_j = C_{ij}$$

25 / 8	2 / 10	ϵ / 7	23 / 6	$U_1 = 0$
12 / -7	9 / -2	40 / 4	7 / -4	$U_2 = -3$
9 / 0	30 / 11	10 / -2	8 / -1	$U_3 = 1$
$V_1 = 8$	$V_2 = 10$	$V_3 = 7$	$V_4 = 6$	

iv) Net evaluations for unoccupied cells:-

$$z_{ij} - C_{ij} = u_i + v_j - C_{ij}$$

v) All $z_{ij} - C_{ij} \leq 0$; Hence we have been reached the optimum solution.

$$\text{Min } Z = 8\times25 + 10\times2 + 7\times0 + 6\times23 + 10\times4 + 30\times11 \qquad (\because \epsilon = 0)$$

$$\text{Min } Z = 848$$

<u>Degeneracy in a transportation Problem</u> :- The number of basic cells must be m+n-1 for an m×n transportation Problem. If the number of basic cells are less than m+n-1, then the solution is degenerate solution. The Problem of degeneracy may occur at any stage (iteration) of solving the transportation Problem.

<u>Case i) Degeneracy at the intial Solution</u> :- To resolve degeneracy at the intial Solution, a very small quantity $\epsilon (>0)$ is allocated in an unoccupied cell so as to get m+n-1 number of occupied cells.

In a minimization transportation Problem it is better to allocate ϵ to unoccupied cells that have lowest ~~temp~~ transportation costs. In some cases ϵ must be added in one of those unoccupied cells which make Possible the determination of u_i and v_j uniquely.

The quantity ϵ is considered to be so small that if it is transfered to an occupied cell, it does not change the quantity of allocation. That is $x_{ij} + \epsilon = x_{ij} - \epsilon = x_{ij}$ but $\epsilon - \epsilon = 0$. Also ϵ does not affect the total transportation cost of the allocation. Hence the quantity ϵ is used to evaluate unoccupied cells and once the purpose is over,

must be removed from the

Case ii) Degeneracy at subsequent iterations :- to resolve degeneracy which occur during optimality test, the quantity ϵ may be allocated to one or more cells which have become unoccupied recently to have m+n -1 number of occupied cells in the new solution. It may be removed once the purpose is over.

Stepping - Stone Method :-

Step1 :- check whether $\Sigma a_i = \Sigma b_j$. If not introduce a dummy origin or dummy destination so that they are equal.

Step2 :- Determine an intial basic feasible solution by using any one of the methods.

Step3 :- check whether the no. of basic cells are m+n-1 or not. If there are less than m+n-1 there exist degeneracy and we introduce a very small positive assignment of $\epsilon (\simeq 0)$ in suitable independent positions, so that the no. of occupied cells is exactly equal to m+n-1.

Step4 :- Select an unoccupied cell. Identify a closed loop which starts and ends at the non-basic cells. The cells at the turning point are called stepping stones. Add and subtract +, - signs alternatively to all the cells of the loop starting with the non-basic cell.

Step5 :- Examine the non-basic cells for a possible decrease in the transportation cost. one unit is allocated to a non-basic cell. Add and subtract the same in the cells of the loop. If the net result of the change is a decrease in the transportation cost, then this solution may be considered as a better solution.

This procedure is done to all non basic cells until there is no further reduction in transportation

-n cost i.e., we examine which non-basic cell give us height reduction in transportation cost. corresponding solution is called an optimum solution.

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UNIT - IV ASSIGNMENT PROBLEM & SEQUENCING PROBLEMS

- ⇒ def of Assignment Problem (AP)
- ⇒ Mathematical formulation of AP
- ⇒ AP as a Particular case of LPP
- ⇒ AP as a Particular case of TP
- ⇒ Balanced and unbalanced AP
- ⇒ Hungarian Method for solving AP
- ⇒ Exceptional cases in AP
  - ⇒ Maximization AP
  - ⇒ Multiple optimum solutions of AP
  - ⇒ Prohibited AP
- ⇒ Travelling Salesman Problem

Sequencing Problems

- ⇒ def of sequencing Problem
- ⇒ Notations and terminology
- ⇒ Assumptions in sequencing Problems.
- ⇒ Processing n- Jobs through two machines
- ⇒ Processing n- Jobs through K machines

Assignment Problem :- The Assignment Problem as a Particular case of TP in which the no. of Jobs are to be assigned to equal no. of persons so that each job has to perform only one person. The objective of AP is to maximize total Profit (or) ~~mini~~ minimize total cost.

Mathematical Formulation of AP :- let us consider n Jobs and n-persons. The assignment Problem is to assign n jobs to n. Persons so, that one Job for each Person and one Person can be given one and only one Job.

The assignment cost matrix can be formulated as follows.

$$
\begin{array}{cc}
 & \text{Persons} \\
 & \begin{array}{ccccccc} 1 & 2 & \cdots & j & \cdots & n & \end{array} \\
\text{Jobs} \begin{array}{c} 1 \\ 2 \\ \vdots \\ i \\ \vdots \\ n \end{array} & \begin{bmatrix}
C_{11} & C_{12} & \cdots & C_{1j} & \cdots & C_{1n} \\
C_{21} & C_{22} & \cdots & C_{2j} & \cdots & C_{2n} \\
\vdots & & & & & \\
C_{i1} & C_{i2} & \cdots & C_{ij} & \cdots & C_{in} \\
\vdots & & & & & \\
C_{n1} & C_{n2} & \cdots & C_{nj} & \cdots & C_{nn}
\end{bmatrix} \begin{array}{c} \text{avai} \\ 1 \\ 1 \\ \vdots \\ 1 \\ \vdots \\ 1 \end{array} \\
\text{Req} & \begin{array}{ccccccc} 1 & 1 & \cdots & 1 & \cdots & 1 & \end{array}
\end{array}
$$

The objective of the Assignment Problem is to assign n persons to n jobs so that minimize the total assignment cost.

let $x_{ij} = \begin{cases} 1 \text{ ; if } i^{th} \text{ job is assigned to } j^{th} \text{ person} \\ 0 \text{ ; otherwise.} \end{cases}$

C_{ij} is cost to assign i^{th} Job to j^{th} Person.

Assignment Problem as a Particular case of LPP :-

The assignment Problem can be mathematically formulated as LPP as follows:

$$\text{Min } Z = \sum_{i=1}^{n}\sum_{j=1}^{n} x_{ij} c_{ij}$$

Subject to the constraints

$$\sum_{i=1}^{n} x_{ij} = 1 \; ; \; j = 1, 2 \ldots n$$

$$\sum_{j=1}^{n} x_{ij} = 1 \; ; \; i = 1, 2, \ldots n$$

and $x_{ij} = 0$ and 1

This formulation of assignment is looking to an LPP.

Assignment Problem as a Particular case of tp :-

An $m \times n$ transportation Problem becomes an Assignment Problem under the following conditions.

1. $m = n$ i.e., no. of origins equal to no. of destinations.
2. All a_i and b_j values are unity

   i.e., $a_i = 1$, $b_j = 1$ $\forall$ i and j
3. The quantity x_{ij} takes the values 0 and 1 for all i and j. This indicates x_{ij} can be unity associated with one origin and one destination.

Applications of Assignment Problem :- The two major applications of assignment Problem are as follows,

i) Assignment Problem is used in assignment of salesman to territories vehicles to routes, Men to Job machines, machines to Jobs etc.

ii) Assignment problem is used in assignment of salesmen to territories.

Assignment Problem (minimization case) HUNGARIAN METHOD

Step1:- Determine the cost table from the given Problem.

i) If the number of sources is equal to the number of destinations, go to step3.

ii) If the no. of sources is not equal to the number of destinations, go to step 2.

Step2:- Add a dummy source or dummy destination, so that the cost table become a square matrix. The cost entries of dummy source/destinations are always zero.

Step3:- Locate the smallest element in each row of the given cost matrix and then subtract the same from each element of that row.

Step4:- In the reduced matrix obtained in step 3, locate the smallest element of each column and then subtract the same from each element of that column. Each column and row now have at least one zero.

Step5:- In the modified matrix obtained in step4, search for an optimal assign-ment as follows:

a) Examine the rows successively until a row with a single zero is found. Enrectangle this zero (□) and cross off (x) all other zeros in its column. Continue in this manner until all the rows have been taken care of.

b) Repeat the procedure for each column of the reduced matrix.

c) If a row and/or column has two or more zeroes and one cannot be chosen by inspection, then assign arbitary any one of these zeros and cross off all other zeroes of that row/column.

d) Repeat (a) through (c) above successively until the chain of assigning (□) or cross(x) ends.

Step6:- If the number of assignments (□) is equal to n (the order of the cost matrix), an optimum solution is reached.

If the number of assignments is less than n (the order of the matrix), go to the next step.

Step 7:- Draw the minimum number of horizontal and/or vertical lines to cover all the zeroes of the reduced matrix. This can be conveniently done by using a simple procedure:

a) Mark (✓) rows that do not have any assigned zero.

b) Mark (✓) column that have zeroes in the marked rows.

c) Mark (✓) rows that have assigned zeroes in the marked columns.

d) Repeat (b) and (c) above until the chain of marking is completed.

e) Draw lines through all the unmarked rows and marked columns. This gives us the desired minimum number of lines.

Step 8:- Develop the new revised cost matrix as follows:

a) Find the smallest element of the reduced matrix not covered by any of the lines.

b) Subtract this element from all the uncovered elements and add the same to all the elements lying at the intersection of any two lines.

Step 9:- Go to Step 6 and repeat the procedure until an optimum solution is attained.

Problems on Assignment Problem

1Q. There are four jobs A, B, C and D these are performed on 4 machines 1, 2, 3, 4 one job is allocated through a machine. The cost of each job on each machine is given below in the following matrix.

$$\begin{array}{c|c} & \text{Machines} \\ \text{Jobs} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 15 & 14 & 12 & 16 \\ 23 & 22 & 25 & 29 \\ 31 & 34 & 32 & 33 \\ 21 & 32 & 44 & 53 \end{bmatrix} \end{array}$$

obtain an optimum assignment schedule?

Sol.:) No. of rows = 4

No. of columns = 4

∴ No. of rows = No. of columns, then go to Step iii.

iii) Row Reduced matrix:-

$$\begin{bmatrix} 3 & 2 & 0 & 4 \\ 1 & 0 & 3 & 2 \\ 0 & 3 & 1 & 2 \\ 0 & 11 & 23 & 32 \end{bmatrix}$$

iv) & v) column Reduced matrix:-

$$\begin{bmatrix} 3 & 2 & \boxed{0} & 2 \\ 1 & \boxed{0} & 3 & \cancel{0} \\ \cancel{0} & 3 & 1 & \boxed{0} \\ \boxed{0} & 11 & 23 & 30 \end{bmatrix}$$

vi) No. of assignments = 4
order of the matrix = 4

∴ No. of Assignment = order of the matrix

Hence we have been reached the optimum sol. The optimum assignments are given by

A → 3(12)
B → 2(22)
C → 4(33)
D → 1(21)
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2A. There are 4 Jobs A, B, C, D and these are performed on 4 machines 1, 2, 3, 4. one Job is allocated to a machine, the cost of each Job on each machine is given below in the following matrix:

$$\begin{array}{cc} & \text{Machines} \\ & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \text{Jobs} & \begin{bmatrix} 10 & 25 & 15 & 20 \\ 15 & 30 & 5 & 15 \\ 35 & 20 & 12 & 24 \\ 17 & 25 & 24 & 20 \end{bmatrix} \end{array}$$

Sol. i) No. of rows = 4

No. of Columns = 4

∴ No. of rows = No. of col., then go to step iii.

iii) Row Reduced matrix :-

$$\begin{bmatrix} 0 & 15 & 5 & 10 \\ 10 & 25 & 0 & 10 \\ 23 & 8 & 0 & 12 \\ 0 & 8 & 7 & 3 \end{bmatrix}$$

iv) col. Reduced matrix & v)

$$\begin{bmatrix} \boxed{0} & 7 & 5 & 7 \\ 10 & 17 & \boxed{0} & 7 \\ 23 & \boxed{0} & \cancel{0} & 9 \\ \cancel{0} & \cancel{0} & 7 & \boxed{0} \end{bmatrix}$$

vi) No. of ~~assignm = 4~~ assignments = 4

order of the matrix = 4

∴ No. of assignments = order of the matrix

Hence we have been reached the optimum Sol. The optimum assignments are given by

A → 1 (10)

B → 3 (5)

C → 2 (20)

D → 4 (20)

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3Q. A Department has five employees with 5 Jobs to be performed. The time in hours of each machine to perform a job is given below. Find the optimum assignment schedule.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 10 & 5 & 13 & 15 & 16 \\ 3 & 9 & 19 & 13 & 6 \\ 10 & 7 & 2 & 2 & 2 \\ 7 & 11 & 9 & 7 & 12 \\ 7 & 9 & 10 & 4 & 12 \end{bmatrix} \end{array}$$

Sol. i) No. of rows = 5

No. of columns = 5

∴ No. of rows = No. of cols, then go to step (iii)

iii) Row Reduced matrix:-

$$\begin{bmatrix} 5 & 0 & 8 & 10 & 11 \\ 0 & 6 & 16 & 10 & 3 \\ 8 & 5 & 0 & 0 & 0 \\ 0 & 4 & 2 & 0 & 5 \\ 3 & 5 & 6 & 0 & 8 \end{bmatrix}$$

iv) Col. Reduced matrix:- & v)

$$\begin{bmatrix} 5 & \boxed{0} & 8 & 10 & 11 \\ \boxed{0} & 6 & 16 & 10 & 3 \\ 8 & 5 & \cancel{0} & \cancel{0} & \boxed{0} \\ \cancel{0} & 4 & 2 & \cancel{0} & 5 \\ 3 & 5 & 6 & \boxed{0} & 8 \end{bmatrix}$$

vi) No. of assignments = 4

order of the matrix = 5

∴ No. of assignments ≠ order of the matrix, then go to step 7)

vii)

$$\begin{bmatrix} 5 & \boxed{0} & 8 & 10 & 11 \\ \boxed{0} & 6 & 16 & 10 & 3 \\ 8 & 5 & \cancel{0} & \cancel{0} & \boxed{0} \\ 0 & 4 & 2 & 0 & 5 \\ 3 & 5 & 6 & \boxed{0} & 8 \end{bmatrix}$$

viii) The Min. element from all the uncovered elements = 2. It should be subtracted from all the uncovered elements and add to the intersection elements, then we get

$$\begin{bmatrix} 7 & \boxed{0} & 8 & 12 & 11 \\ \boxed{0} & 4 & 14 & 10 & 1 \\ 10 & 5 & \cancel{0} & 2 & \boxed{0} \\ \cancel{0} & 2 & \boxed{0} & \cancel{0} & 3 \\ 3 & 3 & 4 & \boxed{0} & 6 \end{bmatrix}$$

No. of assignments = 5

order of the matrix = 5

No. of assignments = order of the matrix. Hence we have been reached the optimum solution. The optimum assignments are given by

A → 2(5)

B → 1(3)

C → 5(2)

D → 3(9)

E → 4(4)

23 hrs

4Q. Find the optimum Sto Solution.

		Mon	G	M
	A	18	26	17	11
Task	B	13	28	14	26
	C	38	19	18	15
	D	19	26	24	10

Sol.i) No. of rows = 4

No. of cols = 4

∴ No. of rows = No. of cols.

iii) Row Reduced matrix :-

$$\begin{bmatrix} 7 & 15 & 6 & 0 \\ 0 & 15 & 1 & 13 \\ 23 & 4 & 3 & 0 \\ 9 & 16 & 14 & 0 \end{bmatrix}$$

iv) Col. Reduced matrix :- & v)

$$\begin{bmatrix} 7 & 11 & 5 & \boxed{0} \\ \boxed{0} & 11 & \cancel{0} & 13 \\ 23 & \boxed{0} & 2 & \cancel{0} \\ 9 & 12 & 13 & \cancel{0} \end{bmatrix}$$

vi) No. of assignments = 3

order of the matrix = 4.

∴ No. of assignments ≠ order of the matrix.

vii)

$$\begin{bmatrix} 7 & 11 & 5 & \boxed{0} \\ \cancel{\boxed{0}} & \cancel{11} & \cancel{\cancel{0}} & \cancel{13} \\ \cancel{23} & \cancel{\boxed{0}} & \cancel{2} & \cancel{\cancel{0}} \\ 9 & 12 & 13 & \cancel{\cancel{0}} \end{bmatrix}$$

viii) The Min. element from all the uncovered elements = 5. It should be sub from all the uncovered elements and add to the intersection elements, then we get.

$$\begin{bmatrix} 2 & 6 & \boxed{0} & 0 \\ \boxed{0} & 11 & 0 & 18 \\ 23 & \boxed{0} & 2 & 5 \\ 4 & 7 & 8 & \boxed{0} \end{bmatrix}$$

∴ No. of assignments = 4

order of the matrix = 4

∴ No. of assignments = order of the matrix. Hence we have been reached the optimum Sol. The optimum Sol. is given by

A → 3 (17)

B → 1 (13)

C → 2 (19)

D → 4 (10)

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5Q. Solve the following Assignment Problem. (Unbalanced Assignment Problem)

Machines

Jobs	A	B	C	D	E
1	6	2	5	2	6
2	2	5	8	7	7
3	7	8	6	9	8
4	6	2	3	4	5
5	9	3	8	9	0
6	4	7	5	6	8

Sol. i) No. of rows = 6

No. of cols = 5

∴ No. of rows ≠ No. of cols., then go to step ii)

ii) It is an unbalanced Assignment Problem, make it balance by introducing a dummy machine 'F'. so that no. of rows is equal to no. of cols, then we get

$$
\begin{array}{c|cccccc}
 & A & B & C & D & E & F \\
\hline
1 & 6 & 2 & 5 & 2 & 6 & 0 \\
2 & 2 & 5 & 8 & 7 & 7 & 0 \\
3 & 7 & 8 & 6 & 9 & 8 & 0 \\
4 & 6 & 2 & 3 & 4 & 5 & 0 \\
5 & 9 & 3 & 8 & 9 & 0 & 0 \\
6 & 4 & 7 & 5 & 6 & 8 & 0
\end{array}
$$

iii) Row Reduced matrix :-

$$
\begin{bmatrix}
6 & 2 & 5 & 2 & 6 & 0 \\
2 & 5 & 8 & 7 & 7 & 0 \\
7 & 8 & 6 & 9 & 8 & 0 \\
6 & 2 & 3 & 4 & 5 & 0 \\
9 & 3 & 8 & 9 & 0 & 0 \\
4 & 7 & 5 & 6 & 8 & 0
\end{bmatrix}
$$

iv) Col. Reduced matrix :- & v.) & vii)

$$
\begin{bmatrix}
4 & \boxed{0} & 2 & \cancel{0} & 6 & \cancel{0} \\
\boxed{0} & 3 & 5 & 5 & 7 & \cancel{0} \\
5 & 6 & 3 & 7 & 8 & \cancel{0} \\
4 & \cancel{0} & \boxed{0} & 2 & 5 & \cancel{0} \\
7 & 1 & 5 & 7 & \boxed{0} & \cancel{0} \\
2 & 5 & 2 & 4 & 8 & \cancel{0}
\end{bmatrix}
$$

vi) No. of assignments = 5

order of the matrix = 6

∴ No. of assignments ≠ order of the matrix, then go to step 7.

viii) The Min. element from all the uncovered elements = 2. It should be subtracted from all the uncovered elements and add to the intersect elements, then we get.

$$\begin{bmatrix} 4 & \cancel{0} & 2 & \boxed{0} & 6 & 2 \\ \boxed{0} & 3 & 5 & 5 & 7 & 2 \\ 3 & 4 & 1 & 5 & 6 & \boxed{0} \\ 4 & \boxed{0} & \cancel{0} & 2 & 5 & 2 \\ 7 & 1 & 5 & 7 & \boxed{0} & 2 \\ \cancel{0} & 3 & \boxed{0} & 2 & 6 & \cancel{0} \end{bmatrix}$$

No. of assignments = 6

order of the matrix = 6

∴ No. of assignments = order of the matrix. Hence we have been reached the optimum sol. The optimum sol. is given by

$1 \rightarrow D(2)$

$2 \rightarrow A(2)$

$3 \rightarrow F(0)$

$4 \rightarrow B(2)$

$5 \rightarrow E(0)$

$6 \rightarrow C(5)$

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<u>Maximization Case in Assignment Problem</u> :- In some cases, the Pay off elements of the assignment Problem may represent revenues or Profits instead of Costs so that the objective will be to maximize the total revenue or Profit. The Hungarian method explained earlier can also be used for maximization case. The Problem of maximization can be converted into a minimization one by selecting the largest element of the Profit matrix and then subtracting from it all the elements of the matrix. This gives us an opportunity loss matrix. We then Proceed as usual and obtain the optimum assignment schedule. Maximum revenue/Profits then obtained by restoring the original values of those cells in which the assignments have been made.

Prohibited Assignments:- Sometimes due to certain reasons, a particular resource (say a man or a machine) cannot be assigned to perform a particular activity (say a territory or a job). In such cases, the cost of performing that particular activity by a particular resource is considered to be very large (written as M or ∞) so as to prohibit the entry of this pair of resource-activity into the final solution.

(Multiple optimum Solutions of Assignment Problem)

1Q. A department in a company has 5 employees, they have to perform 5 jobs, the time in hours that each employ take to perform each job is given in the following matrix. Determine an optimum allocation of allocating of one job to one employ with minimum cost.

Jobs

$$\begin{array}{c|c} & \begin{array}{ccccc} I & II & III & IV & V \end{array} \\ \text{Employees} \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \begin{bmatrix} 1 & 3 & 2 & 3 & 6 \\ 2 & 4 & 3 & 1 & 5 \\ 5 & 6 & 3 & 4 & 6 \\ 3 & 1 & 4 & 2 & 2 \\ 1 & 5 & 6 & 5 & 4 \end{bmatrix} \end{array}$$

Sol. i) No. of rows = 5

No. of rows = 5

$\therefore$ No. of rows = No. of rows.

) Row Reduced matrix :-

$$\begin{bmatrix} 0 & 2 & 1 & 2 & 5 \\ 1 & 3 & 2 & 0 & 4 \\ 2 & 3 & 0 & 1 & 3 \\ 2 & 0 & 3 & 1 & 1 \\ 0 & 4 & 5 & 4 & 3 \end{bmatrix}$$

iv) Col. Reduced matrix :- & v)

$$\begin{bmatrix} \boxed{0} & 2 & 1 & 2 & 4 \\ 1 & 3 & 2 & \boxed{0} & 3 \\ 2 & 3 & \boxed{0} & 1 & 2 \\ 2 & \boxed{0} & 3 & 1 & \cancel{0} \\ \cancel{0} & 4 & 5 & 4 & 2 \end{bmatrix}$$

) No. of assignments = 4

order of the matrix = 5

No. of assig. $\neq$ order of the matrix.

vii)

$$\begin{bmatrix} \boxed{0} & 2 & 1 & 2 & 4 \\ 1 & 3 & 2 & \boxed{0} & 3 \\ 2 & 3 & \boxed{0} & 1 & 2 \\ 2 & \boxed{0} & 3 & 1 & 0 \\ 0 & 4 & 5 & 4 & 2 \end{bmatrix}$$

viii) The min. element from all the uncovered elements = 1. It should be subtracted from all the uncovered elements and add to the intersection elements, then we get

$$\begin{bmatrix} \boxed{0} & 1 & 0 & 1 & 3 \\ 2 & 3 & 2 & \boxed{0} & 3 \\ 3 & 3 & \boxed{0} & 1 & 2 \\ 3 & \boxed{0} & 3 & 1 & 0 \\ 0 & 3 & 4 & 3 & 1 \end{bmatrix}$$

No. of assig = 4

order of the matrix = 5

∴ No. of assig ≠ order of the matrix.

$$\begin{bmatrix} \boxed{0} & 1 & 0 & 1 & 3 \\ 2 & 3 & 2 & \boxed{0} & 3 \\ 3 & 3 & \boxed{0} & 1 & 2 \\ 3 & \boxed{0} & 3 & 1 & 0 \\ 0 & 3 & 4 & 3 & 1 \end{bmatrix}$$

The min. element from all the uncovered elements = 1. It should be sub from all the uncovered elements and add to the intersection elements, then we get

P.T.O

$$\begin{bmatrix} \boxed{0} & \cancel{0} & \cancel{0} & \cancel{0} & 2 \\ 3 & 3 & 3 & \boxed{0} & 3 \\ 3 & 2 & \boxed{0} & \cancel{0} & 1 \\ 4 & \boxed{0} & 4 & 1 & \cancel{0} \\ \cancel{0} & 2 & 4 & 2 & \boxed{0} \end{bmatrix} \text{ or } \begin{bmatrix} \cancel{0} & \boxed{0} & \cancel{0} & \cancel{0} & 2 \\ 2 & 3 & 3 & \boxed{0} & 3 \\ 2 & 2 & \boxed{0} & \cancel{0} & 1 \\ 4 & \cancel{0} & 4 & 1 & \boxed{0} \\ \boxed{0} & 2 & 4 & 2 & 0 \end{bmatrix}$$

No. of assignments = 5

Order of the matrix = 5

$A \to 2(3)$
$B \to 4(1)$
$C \to 3(3)$
$D \to 5(2)$
$E \to 1(1) = 10$

No. of assignments = order of the matrix. Hence we have reached the optimum solution. The optimum sol. is given by. Therefore this assignment has multiple sol.

$A \to 1(1)$

$B \to 4(1)$

$C \to 3(3)$

$D \to 2(1)$

$E \to 5(4)$

$\overline{10}$

30/4/22 <u>Maximization Assignment Problems</u> :-

1Q. A company has four salesman targeted at four cities. The Profits per day in rupees for each salesman in each city is given below. Find the assignment of salesman to various cities which maximises the total Profit.

$$\begin{array}{cc} & \text{Salesman} \\ & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \text{Cities} \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{bmatrix} 16 & 10 & 14 & 11 \\ 14 & 11 & 15 & 15 \\ 15 & 15 & 13 & 12 \\ 13 & 12 & 14 & 15 \end{bmatrix} \end{array}$$

Sol. The given Problem is a maximization assignment Problem. convert the maximization into minimization by taking the highest value in the cost matrix i.e., 16. Now subtract every value from this maximum value, then we get

$$\begin{bmatrix} 0 & 6 & 2 & 5 \\ 2 & 5 & 1 & 1 \\ 1 & 1 & 3 & 4 \\ 3 & 4 & 2 & 1 \end{bmatrix}$$

No. of rows = 4

No. of cols = 4

∴ No. of rows = No. of columns.

Row Reduced matrix :-

$$\begin{bmatrix} 0 & 6 & 2 & 5 \\ 1 & 4 & 0 & 0 \\ 0 & 0 & 2 & 3 \\ 2 & 3 & 1 & 0 \end{bmatrix}$$

Column Reduced matrix :-

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} \boxed{0} & 6 & 2 & 5 \\ 1 & 4 & \boxed{0} & \cancel{0} \\ \cancel{0} & \boxed{0} & 2 & 3 \\ 2 & 3 & 1 & \boxed{0} \end{bmatrix} \end{array}$$

No. of assignments = 4

order of the matrix = 4

∴ No. of assignments = order of the matrix. Hence we have been the optimum solution. The optimum solution is given by

A → 1 (16)

B → 3 (15)

C → 2 (15)

D → 4 (15)

61

∴ Maximum Profit = Rs 61.

Bq. A Company has 6 Jobs to be processed by 6 machines. The returns in rupees for each job to each machine are given below. Find the assignment so as to maximize the returns of Company.

	J_1	J_2	J_3	J_4	J_5	J_6
M_1	9	22	58	11	19	27
M_2	43	78	72	50	63	48
M_3	41	28	91	37	45	33
M_4	74	42	27	49	39	32
M_5	36	11	57	22	25	18
M_6	13	56	53	31	17	28

Sol. The given Problem is a maximization assignment Problem. Convert the maximization into ~~mini~~ minimization by taking the highest value in the cost matrix i.e., 91. Now subtracting every value from this maximum value, then we get.

$$\begin{bmatrix} 82 & 69 & 33 & 80 & 72 & 64 \\ 48 & 13 & 91 & 41 & 28 & 43 \\ 50 & 63 & 0 & 54 & 46 & 58 \\ 17 & 49 & 64 & 42 & 52 & 59 \\ 55 & 80 & 34 & 69 & 66 & 73 \\ 78 & 35 & 38 & 60 & 74 & 63 \end{bmatrix}$$

No. of rows = 6

No. of cols = 6

$\therefore$ No. of rows = No. of cols.

Row Reduced matrix :-

$$\begin{bmatrix} 49 & 36 & 0 & 47 & 39 & 31 \\ 35 & 0 & 6 & 28 & 15 & 30 \\ 50 & 63 & 0 & 54 & 46 & 58 \\ 0 & 32 & 47 & 25 & 35 & 42 \\ 21 & 46 & 0 & 35 & 32 & 39 \\ 43 & 0 & 3 & 25 & 39 & 28 \end{bmatrix}$$

Col. Reduced matrix :-

$$\begin{bmatrix} 49 & 36 & \boxed{0} & 22 & 24 & 3 \\ 35 & \boxed{0} & 6 & 3 & 0 & 2 \\ 50 & 63 & 0 & 29 & 31 & 30 \\ \boxed{0} & 32 & 47 & 0 & 20 & 14 \\ 21 & 46 & 0 & 10 & 17 & 11 \\ 43 & 0 & 3 & \boxed{0} & 24 & 0 \end{bmatrix}$$

No. of Assignments = 4

order of the matrix = 6

$\therefore$ No. of assignments $\neq$ order of the matrix.

The Min. element from all the uncovered elements = 3. It should be subtracted from all the uncovered elements and add to the intersection elements, then we get

$$\begin{bmatrix} 46 & 33 & 0 & 10 & 21 & \boxed{0} \\ 35 & \boxed{0} & 9 & 3 & 0 & 2 \\ 47 & 63 & \boxed{0} & 26 & 28 & 27 \\ \boxed{0} & 32 & 50 & 0 & 20 & 14 \\ 18 & 43 & 0 & 7 & 14 & 9 \\ 43 & 0 & 6 & \boxed{0} & 24 & 0 \end{bmatrix}$$

No. of Assignments = 5

order of the matrix = 6

$\therefore$ No. of assignments $\neq$ order of the matrix.

The Min. element from all the uncovered elements = 7. It should be subtracted from all the uncovered elements and add to the intersection elements, then we get

$$\begin{bmatrix} 46 & 33 & 7 & 10 & 21 & \boxed{0} \\ 35 & \cancel{0} & 16 & 3 & \boxed{0} & 2 \\ 40 & 56 & \boxed{0} & 19 & 21 & 20 \\ \boxed{0} & 32 & 57 & \cancel{0} & 20 & 14 \\ 11 & 36 & \cancel{0} & \boxed{0} & 7 & 2 \\ 43 & \boxed{0} & 13 & \cancel{0} & 24 & \cancel{0} \end{bmatrix}$$

No. of Assignments = 6

& order of the matrix = 6.

∴ No. of assignments = order of the matrix. Hence we have been reached the optimum solution. The optimum solution is given by

$M_1 \rightarrow J_6$ (27)

$M_2 \rightarrow J_5$ (63)

$M_3 \rightarrow J_3$ (91)

$M_4 \rightarrow J_1$ (74)

$M_5 \rightarrow J_4$ (22)

$M_6 \rightarrow J_2$ (56)

333

∴ optimum Profit = 333

<u>Travelling Salesman Problem</u> :- To explain this Problem, Suppose a salesman has to visit n cities starting from a Particular city. He visit each city only once and return to his starting Point. The objective is to select the sequence of visiting the cities in such a way that his total travelling time is minimised. If Salesman starts from city 1, then he will have $(n-1)!$ different Possible ways of visits (since the remaining cities 2, 3, ... n are to be visited).

Mathematical formulation of the Travelling Salesman Problem :-

let $x_{ij} = \begin{cases} 1, & \text{if salesman travels from city } i \text{ to } j \ (i \neq j) \\ 0, & \text{otherwise} \end{cases}$

let d_{ij} is the distance b/w city i and city j.

since each city must be visited only once, we have

$$\sum_{i=1}^{n} x_{ij} = 1, \ j = 1, 2, \ldots, n, \ i \neq j$$

since the salesman has to leave each city, we have

$$\sum_{j=1}^{n} x_{ij} = 1, \ i = 1, 2 \ldots, n, \ i \neq j$$

The objective function is

$$\text{minimize} \quad z = \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij} x_{ij}$$

and obviously we have $x_{ij} \geq 0$ since they take values 0 and 1.

Formulation of Travelling Salesman Problem as Assignment Problem :-

The objective is to minimize the distance of the travel, i.e., to find an optimum route i.e., minimize $z = \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij} x_{ij}$

x_{ij} must be chosen that no city is visited twice before the completion of tour and he should not travel from city i to i.

$\therefore \ d_{ii} \to \infty$ i.e., $d_{ij} \to \infty$ for $i = j$

and d_{ij} ; d_{ji} need not be equal.

all d_{ij}'s must be non-negative

i.e., $d_{ij} \geq 0$

and $d_{ij} + d_{jk} \geq d_{ik} \ \forall \ i, j, k$.

The cost matrix of the travelling salesman Problem can be formulated as follows:

$$\begin{array}{cc} & \text{TO} \\ \text{From} & \begin{array}{c} \\ 1 \\ 2 \\ \vdots \\ n \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & \cdots & n \end{array} \\ \begin{bmatrix} \infty & d_{12} & \cdots & d_{1n} \\ d_{21} & \infty & \cdots & d_{2n} \\ \vdots & \vdots & & \vdots \\ d_{n1} & d_{n2} & \cdots & \infty \end{bmatrix} \end{array} \end{array}$$

6/5/22

Prohibited Assignment Problem :-

1Q. In the modification of a plant layout of a factory four new machines M_1, M_2, M_3 and M_4 are to be installed in a machine shop. There are five vacant places A, B, C, D and E available. Because of limited space, machine M_2 cannot be placed at C and M_1 cannot be placed at A. The cost of locating of machine i to place j in rupees is shown below. Find the optimum assignment schedule.

Machine \ Location	A	B	C	D	E
M_1	9	11	15	10	11
M_2	12	9	-	10	9
M_3	-	11	14	11	7
M_4	14	8	12	7	8

Sol. The given Problem is a Prohibited assignment Problem, assign 'M' or '∞' to the Prohibited assignment, then apply Hungarian method, then the modified assignment Problem is

$$\begin{bmatrix} 9 & 11 & 15 & 10 & 11 \\ 12 & 9 & M & 10 & 9 \\ M & 11 & 14 & 11 & 7 \\ 14 & 8 & 12 & 7 & 8 \end{bmatrix}$$

No. of rows = 4

No. of cols = 5

∴ No. of rows ≠ No. of cols, it is an unbalanced assignment Problem make it balance by introducing a dummy machine so that no. of row = No. of cols, then the modified assignment Problem is given by

$$\begin{bmatrix} 9 & 11 & 15 & 10 & 11 \\ 12 & 9 & M & 10 & 9 \\ M & 11 & 14 & 11 & 7 \\ 14 & 8 & 12 & 7 & 8 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Row Reduced matrix :-

$$\begin{bmatrix} 0 & 2 & 6 & 1 & 2 \\ 3 & 0 & M & 1 & 0 \\ M & 4 & 7 & 3 & 0 \\ 7 & 1 & 5 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

col. Reduced matrix :-

$$\begin{bmatrix} \boxed{0} & 2 & 6 & 1 & 2 \\ 3 & \boxed{0} & M & 1 & \cancel{0} \\ M & 4 & 7 & 3 & \boxed{0} \\ 7 & 1 & 5 & \boxed{0} & 1 \\ \cancel{0} & \cancel{0} & \boxed{0} & \cancel{0} & \cancel{0} \end{bmatrix}$$

No. of assignment = 5

order of the matrix = 5

∴ No. of assignments = order of the matrix. Hence we have been reached the optimum solution and the optimum solution is given by

$M_1 \rightarrow A(9)$

$M_2 \rightarrow B(9)$

$M_3 \rightarrow E(7)$

$M_4 \rightarrow D(7)$

$M_E \rightarrow C(0)$

32

Problems on travelling Salesmen Problem:-

1Q. A Surveyor has to visit five cities A, B, C, D and E. The distance (in hundred kms) between the five cities are as follows:

$$\begin{array}{c|ccccc} & & & \text{TO} & & \\ & A & B & C & D & E \\ \hline \text{From} \; A & \infty & 4 & 7 & 3 & 4 \\ B & 4 & \infty & 6 & 3 & 4 \\ C & 7 & 6 & \infty & 7 & 5 \\ D & 3 & 3 & 7 & \infty & 7 \\ E & 4 & 4 & 5 & 7 & \infty \end{array}$$

If the Surveyer starts from the city A and has to come back to city A, which route should he selected so that total distance travelled by him is minimum?

Sol. No. of rows = 5

No. of cols = 5

∴ No. of rows = No. of cols.

Row Reduced matrix:-

$$\begin{bmatrix} \infty & 1 & 4 & 0 & 1 \\ 1 & \infty & 3 & 0 & 1 \\ 2 & 1 & \infty & 2 & 0 \\ 0 & 0 & 4 & \infty & 4 \\ 0 & 0 & 1 & 3 & \infty \end{bmatrix}$$

Column Reduced matrix:-

$$\begin{bmatrix} \infty & 1 & 3 & \boxed{0} & 1 \\ 1 & \infty & 2 & \cancel{0} & 1 \\ \cancel{2} & \cancel{1} & \cancel{\infty} & \cancel{2} & \boxed{0} \\ \boxed{0} & \cancel{0} & \cancel{3} & \cancel{\infty} & \cancel{4} \\ \cancel{0} & \boxed{0} & \cancel{0} & \cancel{3} & \cancel{\infty} \end{bmatrix}$$

No. of assignments = 4

order of the matrix = 5

∴ No. of assignments ≠ order of the matrix.

The Min. element from all the uncovered elements = 1. It should be subtracted from all the uncovered elements and add to the intersection elements, then we get

$$\begin{bmatrix} \infty & \boxed{0} & 2 & \cancel{0} & \cancel{0} \\ \cancel{0} & \infty & 1 & \boxed{0} & \cancel{0} \\ 2 & 1 & \infty & 3 & \boxed{0} \\ \boxed{0} & \cancel{0} & 3 & \infty & 4 \\ \cancel{0} & \cancel{0} & \boxed{0} & 4 & \infty \end{bmatrix}$$

No. of assignment = 5

order of the matrix = 5

This is not a travelling salesmen Problem because the salesmen has not visited the cities C & E (A→B→D→A). Consider the next min. element in the cost matrix i.e., ~~equal to~~ = 1. But it occurs twice at the cell no. (2,3) & (3,2)

Case i) shift the assignment from (2,4) to (2,3) then we get.

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} \infty & \cancel{0} & 2 & \boxed{0} & \cancel{0} \\ \cancel{0} & \infty & \boxed{1} & \cancel{0} & \cancel{0} \\ 2 & 1 & \infty & 3 & \boxed{0} \\ \cancel{0} & \boxed{0} & 3 & \infty & 4 \\ \boxed{0} & \cancel{0} & \cancel{0} & 4 & \infty \end{bmatrix} \end{array}$$

A→D→B→C→E→A

3 + 3 + 6 + 5 + 4 = 21

Case ii) shift the assignment from (3,5) to (3,2) then we get

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} \infty & \cancel{0} & 2 & \cancel{0} & \boxed{0} \\ \cancel{0} & \infty & 1 & \boxed{0} & \cancel{0} \\ 2 & \boxed{1} & \infty & 3 & \cancel{0} \\ \boxed{0} & \cancel{0} & 3 & \infty & 4 \\ \cancel{0} & \cancel{0} & \boxed{0} & 4 & \infty \end{bmatrix} \end{array}$$

A→E→C→B→D→A

4 + 5 + 6 + 3 + 3 = 21

5/5/22

2Q. Solve the following travelling salesman Problem.

$$\begin{array}{c|ccccc} & A & B & C & D & E \\ \hline A & \infty & 2 & 5 & 7 & 1 \\ B & 6 & \infty & 3 & 8 & 2 \\ C & 8 & 7 & \infty & 4 & 7 \\ D & 12 & 4 & 6 & \infty & 5 \\ E & 1 & 3 & 2 & 8 & \infty \end{array}$$

Sol. No. of rows = 5

No. of cols = 5

∴ No. of rows = No. of cols.

Row Reduced matrix :-

$$\begin{bmatrix} \infty & 1 & 4 & 6 & 0 \\ 4 & \infty & 1 & 6 & 0 \\ 4 & 3 & \infty & 0 & 3 \\ 8 & 0 & 2 & \infty & 1 \\ 0 & 2 & 1 & 7 & \infty \end{bmatrix}$$

Colu. Reduced matrix :-

$$\begin{array}{c|ccccc} & A & B & C & D & E \\ \hline A & \infty & 1 & 3 & 6 & \boxed{0} \\ B & 4 & \infty & \boxed{0} & 6 & \emptyset \\ C & 4 & 3 & \infty & \boxed{0} & 3 \\ D & 8 & \boxed{0} & 1 & \infty & 1 \\ E & \boxed{0} & 2 & \emptyset & 7 & \infty \end{array}$$

No. of assig. = 5

order of the matrix = 5

This is not a travelling salesmen Problem because the salesmen has not visited the cities B, C & D (A→E→A) Consider the min. element in the cost matrix i.e., =1 But is thrice at the cell no (1,2), (4,3) & (4,5)

Case i) shift the assignment from (1,5) to (1,2)

$$\begin{array}{c|ccccc} & A & B & C & D & E \\ \hline A & \infty & \boxed{1} & 3 & 6 & \emptyset \\ B & 4 & \infty & \boxed{0} & 6 & \emptyset \\ C & 4 & 3 & \infty & \boxed{0} & 3 \\ D & 8 & \emptyset & 1 & \infty & \boxed{1} \\ E & \boxed{0} & 2 & \emptyset & 7 & \infty \end{array}$$

A→B→C→D→E→A

2+3+4+5+1=15

Case ii) shift the assignment from (4,2) to (4,3)

$$\begin{array}{c|ccccc} & A & B & C & D & E \\ \hline A & \infty & \boxed{1} & 3 & 6 & \emptyset \\ B & 4 & \infty & \emptyset & 6 & \boxed{0} \\ C & 4 & 3 & \infty & \boxed{0} & 3 \\ D & 8 & \emptyset & \boxed{1} & \infty & 1 \\ E & \boxed{0} & 2 & \emptyset & 7 & \infty \end{array}$$

A→B→E→A (fails)

Case iii) shift the assignment from (4,2) to (4,5)

$$\begin{array}{c|ccccc} & A & B & C & D & E \\ \hline A & \infty & \boxed{1} & 3 & 6 & \cancel{0} \\ B & 4 & \infty & \boxed{0} & 6 & \cancel{0} \\ C & 4 & 3 & \infty & \boxed{0} & 3 \\ D & 8 & \cancel{0} & 1 & \infty & \boxed{1} \\ E & \boxed{0} & 2 & \cancel{0} & 7 & \infty \end{array}$$

$A \to B \to C \to D \to E \to A$

$2+3+4+5+1 = 15$

3Q. Solve the following travelling Salesmen Problem.

$$\begin{array}{c|ccccc} & A & B & C & D & E \\ \hline A & \infty & 3 & 6 & 2 & 3 \\ B & 3 & \infty & 5 & 2 & 3 \\ C & 6 & 5 & \infty & 6 & 4 \\ D & 2 & 2 & 6 & \infty & 6 \\ E & 3 & 3 & 4 & 6 & \infty \end{array}$$

Sol. No. of rows = 5

No. of cols = 5

∴ No. of rows = No. of cols.

Row Reduced matrix :-

$$\begin{array}{c|ccccc} & A & B & C & D & E \\ \hline A & \infty & 1 & 4 & 0 & 1 \\ B & 1 & \infty & 3 & 0 & 1 \\ C & 2 & 1 & \infty & 2 & 0 \\ D & 0 & 0 & 4 & \infty & 4 \\ E & 0 & 0 & 1 & 3 & \infty \end{array}$$

Colu. Reduced matrix :-

$$\begin{array}{c|ccccc} & A & B & C & D & E \\ \hline A & \infty & 1 & 3 & \boxed{0} & 1 \\ B & 1 & \infty & 2 & \cancel{0} & 1 \\ C & \cancel{2} & \cancel{1} & \cancel{\infty} & \cancel{2} & \boxed{0} \\ D & \cancel{\boxed{0}} & \cancel{0} & \cancel{3} & \cancel{\infty} & \cancel{4} \\ E & \cancel{0} & \boxed{0} & \cancel{0} & \cancel{3} & \cancel{\infty} \end{array}$$

No. of assignments = 4

order of the matrix = 5

∴ No. of assignments ≠ order of the matrix.

The Min. element from all the uncovered elements = 1. It should be subtracted from all the uncovered elements and add to the intersection elements, then we get,

$$\begin{array}{c|ccccc} & A & B & C & D & E \\ \hline A & \infty & \boxed{0} & 2 & \emptyset & \emptyset \\ B & \emptyset & \infty & 1 & \boxed{0} & \emptyset \\ C & 2 & 1 & \infty & 3 & \boxed{0} \\ D & \boxed{0} & \emptyset & 3 & \infty & 4 \\ E & \emptyset & \emptyset & \boxed{0} & 4 & \infty \end{array}$$

No of assignments = 5

Order of the matrix = 5

This is ~~an~~ not a travelling salesmen Problem because the salesmen has not visited the cities C & E (A → B → D → A) Consider the next min. element ie = 1. But it occurs twice at the cell no. (2,3) & (3,2)

i) shift the assignment from (2,4) to (2,3)

$$\begin{array}{c|ccccc} & A & B & C & D & E \\ \hline A & \infty & \emptyset & 2 & \boxed{0} & \emptyset \\ B & \emptyset & \infty & \boxed{1} & \emptyset & \emptyset \\ C & 2 & 1 & \infty & 3 & \boxed{0} \\ D & \emptyset & \boxed{0} & 3 & \infty & 4 \\ E & \boxed{0} & \emptyset & \emptyset & 4 & \infty \end{array}$$

A → D → B → C → E → A

2 + 2 + 5 + 4 + 3 = 16

ii) shift the assignment from (3,5) to (3,2)

$$\begin{array}{c|ccccc} & A & B & C & D & E \\ \hline A & \infty & \emptyset & 2 & \emptyset & \boxed{0} \\ B & \emptyset & \infty & 1 & \boxed{0} & \emptyset \\ C & 2 & \boxed{1} & \infty & 3 & \emptyset \\ D & \boxed{0} & \emptyset & 3 & \infty & \emptyset 4 \\ E & \emptyset & \emptyset & \boxed{0} & 4 & \infty \end{array}$$

A → E → C → B → D → A

3 + 4 + 5 + 2 + 2 = 16

5/5/26

Sequencing Problems :- The selection of an appropriate order for a series of jobs to be done on a finite number of service facilities, in some pre-assigned order, is called sequencing.

Assumptions in Sequencing Problem :-

1. Each job, once started on a machine is to be performed upto completion on that machine.
2. The processing time on each machine is known. Such a time is independent of the order of jobs in which they are to be processed.
3. The time taken by each job in changing over from one machine to another is negligible.
4. A job starts on the machines as soon as the job and the machine both are idle and job is next to machine and the machine is also next to the job.
5. No machine may process more than one job simultaneously.
6. The order of completion of job has no significance.

The order of completion of jobs is independent of sequence of jobs.

Basic terms used in Sequencing :-

1. **Number of Machines** :- It refers to the number of service facilities through which a job must pass before it is assumed to be completed.
2. **Processing order** :- It refers to the order (sequence) in which given machines are required for completing the job.
3. **Processing time** :- It indicates the time required by a job on each machine.
4. **Total elapsed time** :- It is the time interval between starting the first job and completing the last job including the idle time (if any) in a particular order by the given set of machines.

5. Idle time of a machine :- It is a time for which a machine does not have a job to process.

6. No passing rule :- It refers to the rule of maintaining the order in which jobs are to be processed on a given machine.

Johnson's algorithm to obtain optimum sequence for n jobs and two machines :-

This method was developed by ~~John~~ Johnson and Bellman. The following steps involved in this method.

Step 1 :- List the jobs along with their processing times in a table as shown below.

Job no	J_1	J_2	J_3	. . . .	J_n
Processing time on machine M_1	t_{11}	t_{12}	t_{13}	. . . .	t_{1n}
on machine M_2	t_{21}	t_{22}	t_{23}	. . . .	t_{2n}

Step 2 :- Examine the rows for processing times on machines M_1 and M_2 and find the smallest processing time in each row i.e., find

$$\text{Min}\{t_{1j}, t_{2j}\} \text{ for all } j$$

Step 3 :- If the smallest processing time is for the first machine M_1, then place the corresponding job in the first available position in the sequence. If it is for the second machine, then place the corresponding job in the last available position in the sequence.

Step 4 :- If there is a tie in selecting the minimum of all the processing times then there may be three situations.

a) Minimum among all processing times is same for the machine i.e.,

$$\text{Min}\{t_{1j}, t_{2j}\} = t_{1k} = t_{2n}$$

b) If the tie for the minimum occurs among processing times t_{1j} on machine M_1 only then select arbitrarily the job to process first.

c) If the tie for the minimum occurs among processing times t_{2j} on machine M_2 then select arbitrarily the job to process last.

Step 5:- Cross off the jobs already assigned and repeat steps steps 1 through 4, placing the remaining jobs next to first (or) next to last, until all the jobs have been assigned.

Step 6:- Calculate idle time for machine M_1 and M_2

a) Idle time for M_1: Total elapsed time − time when the last job in a sequence finishes on M_1.

b) Idle time for M_2: Time at which the first job in a sequence finishes on $M_1 + \sum_{j=2}^{n}$ (time when j^{th} job in a sequence starts on M_2 − [time when $(j-1)^{th}$ job in a sequence finishes on M_2].

Step 7:- The total elapsed time to process all jobs through two machines is given by total elapsed time = time when the nth job in a sequence finishes on machine M_2

$$= \sum_{j=1}^{n} t_{2j} + \sum_{j=1}^{n} I_{2j}$$

where t_{2j} = Time required for processing j^{th} job on machine M_2.

I_{2j} = Time for which machine M_2 remains idle after processing $(j-1)^{th}$ job and before starting work on j^{th} job.

Processing n Jobs through K Machines:-

Suppose there are n jobs, each of which is to be processed through K machines say $M_1, M_2 \ldots M_K$ in the order $M_1 M_2 \ldots M_K$. The list of jobs with their processing time is.

Job No	1	2	. . .	n
Processing times	M_1	t_{11}	t_{12}	t_{1n}
	M_2	t_{21}	t_{22}	t_{2n}
	⋮	⋮		⋮
	M_K	t_{K1}	t_{K2}	t_{Kn}

Johnson's iterative procedure to obtain an optimum sequence can be summarized as follows.

Step 1:- Find Min t_{1j}, min t_{Kj} and maximum of each of $t_{2j}, t_{3j} \ldots t_{(K-1)j} \ldots$ for all $j = 1, 2 \ldots n$.

Step 2:- check the following conditions:

a) Min $t_{ij} \geq$ Max t_{ij} for $i = 2, 3, \ldots K-1$

(OR)

b) Min $t_{Kj} \geq$ Max t_{ij} for $i = 2, 3 \ldots K-1$

Step 3:- If the in-equalities of step 2 are not satisified, method fails. otherwise go to next step.

Step 4:- Convert the K machine problem into two machine sequencing problem by introducing two fictitious machines G and H such that

$$t_{Gj} = t_{1j} + t_{2j} + \ldots + t_{(K-1)j} \text{ and}$$

$$t_{Hj} = t_{2j} + t_{3j} + \ldots + t_{Kj}$$

Step 5:- Find the optimum sequence for the n jobs and 2 machines equivalent sequencing problem with the prescribed order GH in the same way. The resulting sequence shall be optimum for the given problem.

Processing of n Jobs through three machines:-

Suppose there are n jobs, each job is processed through three machines M_1, M_2 and M_3 in the order M_1, M_2, M_3. let us consider the processing times of the jobs on the three machines M_1, M_2 and M_3 as given.

Job no		1	2	. . . .	n
Processing times	M_1	t_{11}	t_{12}	. . . .	t_{1n}
	M_2	t_{21}	t_{22}	. . . .	t_{2n}
	M_3	t_{31}	t_{32}	. . . .	t_{3n}

Johnson's iterative procedure to obtain an optimum sequence can be summarized in the following steps:

Step1. Find Min t_{1j}, Min t_{3j} and Max t_{2j} for $j = 1, 2 \ldots n$

Step2. check whether

a) Min $t_{1j} \geq$ Max t_{2j} $j = 1, 2 \ldots n$

(or)

b) Min $t_{3j} \geq$ Max t_{2j} $j = 1, 2 \ldots n$

Step3. If the in-equalities of step2 are not satisified, method fails. otherwise go to next step.

Step4. Convert the three machine problem into 2 machine problem by introducing two fictitious machines G and H such that

$$t_{Gj} = t_{1j} + t_{2j} \text{ and}$$

$$t_{Hj} = t_{2j} + t_{3j}$$

Step5. Find the optimum sequence of the jobs on the machines G and H using as Johnson's optimum sequence algorithm of n jobs and two machines. The resulting sequence shall be optimal for the given problem.

9/5/22

Problems on n-Jobs K Machines:-

1Q. We have six jobs, each of which must go through machines A, B and C in the order ABC. The Processing times (in hours) are given in the following table.

Jobs	1	2	3	4	5	6
Machine A	12	10	9	14	7	9
Machine B	7	6	6	5	4	4
Machine C	6	5	6	4	2	4

obtain the optimum job sequence and minimum total elapsed time.

Sol. Given that $n = 6$, $K = 3$

Given Problem is a 6 jobs three machine Problem convert into 6 jobs two machine Problem by using the following conditions.

$Min(t_{1j}) = 7$

$Min(t_{3j}) = 2$

$Max(t_{2j}) = 7$

$Min\ t_{1j}(7) \geq Max\ t_{2j}(7)$, satisfies

(or)

$Min\ t_{3j}(2) \geq Max\ t_{2j}(7)$, fails

Introduce two new machines 'G' & 'H', such that $G = t_{1j} + t_{2j}$ & $H = t_{2j} + t_{3j}$ then the modified sequencing Problem is given by

Jobs	1	2	3	4	5	6
G	19	16	15	19	11	13
H	13	11	12	9	6	8

The sequence order to the given 6 jobs 2 machine Problem is

1	3	2	4	6	5

The total elapsed time and the Idle times for the machines A, B & C can be calculated in the following table.

Jobs	3	5
M-A	9	10
M-B	7	4

The minimum time is 4 at job 5 on machine B, so Process job 5 before Job 1. Now the next minimum Processing time is 7 at Job 3 on machine B, so Process Job 3 before Job 5.

2	4	3	5	1

This is the optimum sequence of the given Problem. The total elapsed time is calculated in the following table.

Jobs	Machine 'A'		Machine 'B'		Idle time for Machine A	Idle time for Machine B
	In	Out	In	Out		
2	0	1	1	7	—	1
4	1	4	7	15	—	—
3	4	13	15	22	—	—
5	13	23	23	27	—	1
1	23	28	28	30	—	1
					0	3 hrs

Idle time for Machine 'A' = (30 − 28) + 0 = 2 hrs

Idle time for Machine 'B' = 3 hrs

Total elapsed time = T = 30 hrs

Eg. A company has six jobs on hand, coded A to F. All the jobs have to go through two machines and I and II. The time required for the jobs on each machine, in hour, is given below.

Jobs	A	B	C	D	E	F
Machine I	1	4	6	3	5	2
Machine II	3	6	8	8	1	5

Draw a sequence table scheduling the six jobs on the two machines and calculate total elapsed time.

Sol.

The min. processing time is 1 corresponding to the jobs A on machine I and E machine II. So process job A first and job E last of the sequence.

A					E

Delete jobs A and E. The reduced problem is

Jobs	B	C	D	F
M-I	4	6	3	2
M-II	6	8	8	5

The Min. processing time is 2 at job F on machine I. So process job F next to job A.

A	F				E

Delete job F and reduced problem is

Jobs	B	C	D
M-I	4	6	3
M-II	6	8	8

The min. processing time is 3 at job D on machine I, so process job 3 next to job F.

A	F	D			E

Jobs	B	C
M-I	4	6
M-II	6	8

The minimum Processing time is 4 at job B on machine I, so Process job B next to job D.

By deleting the job D, the next minimum Processing time is 6 at job C on machine I, therefore Process job C next to job B.

∴ The optimum job sequence is

A	F	D	B	C	E

The total elapsed time is calculated in the following table.

Jobs	Machine-I		Machine-II		Idle time for Machine-I	Idle time for Machine-II
	In	out	In	out		
A	0	1	1	4	–	1
F	1	3	4	9	–	–
D	3	6	9	17	–	–
B	6	10	17	23	–	–
C	10	16	23	31	–	–
E	16	21	31	32	–	–
					0	1 hrs

Idle time for Machine-I = (32−21)+0 = 11 hrs

Idle time for Machine-II = 1 hrs

Total elapsed time T = 32 hrs

3Q. Find the sequence that minimizes the total elapsed time (in hours) required to complete the following tasks on two machines.

Tasks	A	B	C	D	E	F	G	H	I
Machine I	2	5	4	9	6	8	7	5	4
Machine II	6	8	7	4	3	9	3	8	11

Sol. The min. Processing time is 2 at task A on machine I, So process task A at first of the sequence.

A								

Delete task A and the reduced Problem is

TASKS	B	C	D	E	F	G	H	I
M - I	5	4	9	6	8	7	5	4
M - II	8	7	4	3	9	3	8	11

The minimum Processing time is 3 corresponding to the tasks E and G both occurs on machine II. Since the corresponding Processing time of task G on machine I is larger than the corresponding Processing time of task E on machine I, task G will be Processed in the last and task E shall be next to it.

A							E	G

Delete tasks E and G, the reduced problem is

TASKS	B	C	D	F	H	I
M-I	5	4	9	8	5	4
M-II	8	7	4	9	8	11

The min. Processing time is 4 corresponding to the tasks C, I on machine I and D on machine II. Since the corresponding Processing time of task I on machine II is larger than the corresponding Processing time of task C on machine II, Process task I after task A and Process task C next to task I. Now Process task D before to task E in the sequence.

A	I	C				D	E	G

Delete the tasks C, D and I, the reduced problem is

TASKS	B	E	H
M - I	5	8	5
M - II	8	9	8

The minimum processing time is 5 at the tasks B and H both on machine I. But here the corresponding processing times of tasks B and H on machine II are equal to 8. Therefore, we can process tasks B and H after task C in two ways, first process B and then H or first process H and then B.

By deleting the tasks B and H, the next minimum processing time is 8 at task F on machine I, so process after B and H or H and B.

∴ The optimum sequence of the tasks is

A	I	C	B	H	F	D	E	G

The min. total elapsed time is calculated in the following table:

TASKS	Machine I		Machine II		Idle time for Machine I	Idle time for Machine II
	In	out	In	out		
A	0	2	2	8	–	2
I	2	6	8	19	–	–
C	6	10	19	26	–	–
B	10	15	26	34	–	–
H	15	20	34	42	–	–
F	20	28	42	51	–	–
D	28	37	51	55	–	–
E	37	43	55	58	–	–
G	43	50	58	61	–	–
					0	2 hrs

Idle time for Machine I $= (61 - 50) + 0 = 11$ hrs

Idle time for Machine II $= 2$ hrs

Total elapsed time $T = 61$ hrs.,

Problems on n- Jobs K Machines:

1Q. We have six jobs, each of which must go through machines A, B and C in the order ABC. The Processing times (in hours) are given in the following table.

Jobs	1	2	3	4	5	6
Machine A	12	10	9	14	7	9
Machine B	7	6	6	5	4	4
Machine C	6	5	6	4	2	4

Obtain the optimum job sequence and minimum total elapsed time.

Sol. Given that $n = 6$, $K = 3$

Given Problem is a 6 jobs three machine Problem convert into 6 jobs two machine Problem by using the following conditions.

$Min(t_{1j}) = 7$

$Min(t_{3j}) = 2$

$Max(t_{2j}) = 7$

$Min\ t_{1j}(7) \geq Max\ t_{2j}(7)$, satisfies

(or)

$Min\ t_{3j}(2) \geq Max\ t_{2j}(7)$, fails

Introduce two new machines 'G' & 'H', such that $G = t_{1j} + t_{2j}$ & $H = t_{2j} + t_{3j}$, then the modified sequencing Problem is given by

Jobs	1	2	3	4	5	6
G	19	16	15	19	11	13
H	13	11	12	9	6	8

The sequence order to the given 6 jobs 2 machine Problem is

1	3	2	4	6	5

The total elapsed time and the Idle times for the machines A, B & C can be calculated in the following table.

Jobs	Machine A		Machine B		Machine C		Idle time for Machine A	Idle time for Machine B	Idle time for Machine C
	In	out	In	out	In	out			
1	0	12	12	19	19	25	–	12	19
3	12	21	21	27	27	33	–	2	2
2	21	31	31	37	37	42	–	4	4
4	31	45	45	50	50	54	–	8	8
6	45	54	54	58	58	62	–	4	4
5	54	61	61	65	65	67	–	3	3
							0	33	40

Total elapsed time = 67 hrs

Idle time for Machine A = 0 + (67 − 61) = 6 hrs

Idle time for Machine B = 33 + (67 − 65) = 35 hrs

Idle time for Machine C = 40 + (67 − 67) = 40 hrs.

2Q. Find the sequence for the following eight jobs that will minimize the total elapsed time for the completion of all jobs. Each job is processed in the same order CAB.

Machines \ Jobs	1	2	3	4	5	6	7	8
A	4	6	7	4	5	3	6	2
B	8	10	7	8	11	8	9	13
C	5	6	2	3	4	9	15	11

Sol. The Given order is CAB. The modified sequencing Problem is given by

Machines \ Jobs	1	2	3	4	5	6	7	8
C	5	6	2	3	4	9	15	11
A	4	6	7	4	5	3	6	2
B	8	10	7	8	11	8	9	13

Given Problem is a 8 jobs three machine problem convert into 8 jobs two machine problem by using the following condition.

$Min(t_{1j}) = 2$

$Min(t_{3j}) = 7$

$Max(t_{2j}) = 7$

$Min\, t_{1j}(2) \geq Max\, t_{2j}(7)$, fails

(or)

$Min\, t_{3j}(7) \geq Max\, t_{2j}(7)$, satisfies.

Introduce two new machines 'R' & 'S', such that $R = t_{1j} + t_{2j}$ & $S = t_{2j} + t_{3j}$, then the modified sequencing problem is given by.

Jobs / Machines	1	2	3	4	5	6	7	8
R	9	12	9	7	9	12	21	13
S	12	16	14	12	16	11	15	15

The sequence order to the given 8 jobs two machine problem is

4	5	3	1	2	8	7	6

The total elapsed time and the idle times for the machines A, B & C can be calculated in the following table.

Jobs	Machine C		Machine B		Machine B		Idle time for M-C	Idle time for M-A	Idle time for M-B
	In	out	In	out	In	out			
4	0	3	3	7	~~15~~ 7	15	–	4	7
5	3	7	7	12	~~26~~ 15	26	–	–	–
3	7	9	12	19	~~33~~ 26	33	–	–	–
1	9	14	19	23	~~41~~ 33	41	–	–	–
2	14	20	23	29	~~51~~ 41	51	–	–	–
8	20	31	31	33	~~64~~ 51	64	–	2	–
7	31	46	46	52	~~73~~ 64	73	–	13	–
6	46	55	55	58	~~81~~ 73	81	–	3	–
							0	22	7

Total elapsed time $T = 81$

Idle time for Machine C = ~~0+(55~~ $0 + (81-55) = 26$ hrs

Idle time for Machine A = $22 + (81-58) = 44$ hrs

Idle time for Machine B = $78 + (81-81) = 78$ hrs

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3Q. Determine the optimal sequence of jobs that minimize the total elapsed time based on the given processing times.

Jobs	1	2	3	4	5	6	7
Machine A	3	8	7	4	9	8	7
Machine B	4	3	2	5	1	4	3
Machine C	6	7	5	11	5	6	12

Sol. Given Problem is a 7 jobs three machine Problem convert it into 7 jobs two machine Problem by using the following conditions.

$\text{Min } t_{1j} = 3$

$\text{Min } t_{3j} = 5$

$\text{Max } t_{2j} = 5$

$\text{Min } t_{1j}(3) \geq \text{Max } t_{2j}(5)$, fails

or

$\text{Min } t_{3j}(5) \geq \text{Max } t_{2j}(5)$, satisfies

Introduce two new machines 'x' & 'y', such that $x = t_{1j} + t_{2j}$ & $y = t_{3j} + t_{2j}$

Jobs	1	2	3	4	5	6	7
x	7	11	9	~~10~~ 9	10	12	10
y	10	10	7	16	6	10	15

The sequence order to the given 7 jobs two machine problem is

1	4	7	2	6	3	5

The total elapsed time and the idle times for the machines A, B & C can be calculated in the following table.

Jobs	Machine A		Machine B		Machine C		Idle time for M-A	Idle time for M-B	Idle time for M-C
	In	out	In	out	In	out			
1	0	3	3	7	7	13	–	3	7
4	3	7	7	12	13	24	–	–	–
7	7	14	14	17	24	36	–	2	–
2	14	22	22	25	36	43	–	5	–
6	22	30	30	34	43	49	–	5	–
3	30	37	37	39	49	54	–	3	–
5	37	46	46	47	54	59	–	7	–
							0	25 hrs	7

Total elapsed time $T = 59$ hrs

Idle time for Machine A $= 0 + (59 - 46) = 13$ hrs

Idle time for Machine B $= 25 + (59 - 47) = 37$

Idle time for Machine C $= 7$ hrs

4Q. Find the sequence order, idle times & total elapsed time to the following data.

Jobs	A	B	C	D	E	F
Machine M_1	8	3	7	2	5	1
Machine M_2	3	4	5	2	1	6
Machine M_3	8	7	6	9	10	9

Sol. $\text{Min } t_{1j} = 1$

$\text{Min } t_{3j} = 6$

$\text{Max } t_{2j} = 6$

$\text{Min } t_{1j}(1) \geq \text{Max } t_{2j}(6)$, fails

or

$\text{Min } t_{3j}(6) \geq \text{Max } t_{2j}(6)$, satisfies

Introduce two new machines 'x' & 'y', such that $x = t_{1j} + t_{2j}$ & $y = t_{2j} + t_{3j}$, then the modified sequencing problem is given by

Jobs	A	B	C	D	E	F
x	11	7	12	4	6	7
y	11	11	11	11	11	15

The sequence order to the given 6 jobs two machine Problem is

D	E	F	B	A	C

The total elapsed time and the Idle times for the machine A, B & C can be calculated in the following table.

Jobs	Machine M_1		Machine M_2		Machine M_3		Idle time for $M-M_1$	Idle time for $M-M_2$	Idle time for $M-M_3$
	In	out	In	out	In	out			
D	0	2	2	4	4	13	–	2	4
E	2	7	7	8	13	23	–	–	–
F	7	8	8	14	23	32	–	–	–
B	8	11	14	18	32	39	–	–	–
A	11	19	19	22	39	47	–	–	–
C	19	26	26	31	47	53	–	–	–
							0	2	4

Total elapsed time $T = 53$ hrs

Idle time for Machine $M_1 = 0 + (53 - 26) = 27$ hrs

Idle time for Machine $M_2 = 2 + (53 - 31) = 24$ hrs

Idle time for Machine $M_3 = 4$ hrs